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Multiphase lattice Boltzmann flux solver for incompressible multiphase flows with large density ratio

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ABSTRACT

A multiphase lattice Boltzmann flux solver (MLBFS) is proposed in this paper for incompressible multiphase flows with low- and large-density-ratios. In the solver, the flow variables at cell centers are given from the solution of macroscopic governing differential equations (Navier–Stokes equations recovered by multiphase lattice Boltzmann (LB) model) by the finite volume method. At each cell interface, the viscous and inviscid fluxes are evaluated simultaneously by local reconstruction of solution for the standard lattice Boltzmann equation (LBE). The forcing terms in the governing equations are directly treated by the finite volume discretization. The phase interfaces are captured by solving the phase-field Cahn–Hilliard equation with a fifth order upwind scheme. Unlike the conventional multiphase LB models, which restrict their applications on uniform grids with fixed time step, the MLBFS has the capability and advantage to simulate multiphase flows on non-uniform grids. The proposed solver is validated by several benchmark problems, such as two-phase co-current flow, Taylor–Couette flow in an annulus, Rayleigh–Taylor instability, and droplet splashing on a thin film at density ratio of 1000 with Reynolds numbers ranging from 20 to 1000. Numerical results show the reliability of the proposed solver for multiphase flows with high density ratio and high Reynolds number.

1. Introduction

Multiphase flows of incompressible fluids are ubiquitous in nature and of great interest in both science and engineering applications [1–4]. To effectively simulate such flows, various numerical methods, including volume of fluid (VOF) method [5], level set approach [6], front tracking method [7] and diffuse interface methods [8,9] have been proposed. Among them, the lattice Boltzmann method (LBM) [10,11] received more and more attention in recent years. As a mesoscopic method with microscopic models, LBM has an instinct kinetic nature and only involves simple algebraic manipulations of streaming-collision processes. Due to these attractive features, many multiphase lattice Boltzmann (LB) models [12–25] have been developed for simulating multiphase flows with low- and large-density-ratios [26–30] during the past two decades.

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The early attempts in the 1990s were focused on the development of reliable multiphase LB models [13–17]. The first model, also known as color-gradient model, was perhaps proposed by Gunstensen et al. [13]. This model introduces an additional two-phase collision step to control interfacial interactions caused by surface tension. Later, Shan and Chen [14,15] developed a potential multiphase LB model for multiphase and multi-component flows by introducing inter-particle interaction forces. Although it cannot satisfy the momentum conservation law at a local position [14], the Shan–Chen (SC) LB model has attained much popularity due to its overall accuracy and ease for implementation. Unlike the color-gradient model, Swift et al. [16] introduced a thermodynamically consistent multiphase LB model based on the free-energy functional, which successfully satisfies the momentum conservation law. Unfortunately, due to the intrinsic density variation across the interface in multiphase flows, this model lacks Galilean invariance which plagues its wide application. He et al. [17] proposed a double-distribution-functions (DDFs) model based on the kinetic theory. One of the DDFs is adopted for the evolution of pressure and velocity field and the other DDF is used to capture the interface of different phases.

Due to numerical instability, these multiphase LB models [13–17] restrict their applications for multiphase flows with low- and/or moderate-density-ratio. There were also several attempts to develop multiphase LB models for multiphase flows with large-density-ratio [18–23] during the past decade. A projection-like LB model [19] was proposed by enforcing the incompressibility condition through a pressure-correction process. This model has been applied for binary flows with density ratio up to 1000. However, the pressure correction step [19] spoils the simplicity and efficiency of the LBM. To get a more effective model, Lee and Lin [20] and Lee and Liu [21] proposed a stable discretization algorithm for LBE on the basis of the DDF model [17]. Their method introduces the thermodynamic pressure tensor in a specific form and directional derivatives to stabilize the solution process. Zheng et al. [22] later proposed an improved LB model, which is able to accurately recover Cahn–Hilliard equation, i.e., the interface capturing equation. Yan and Zu [23] combined the projection-like LB model [19] and the free energy model [24,25] to simulate multiphase flows with large density ratio and partial wetting surfaces.

The multiphase LB models mentioned above [12–25] share the same advantages of LBM, such as intrinsic kinetic nature, simple streaming and collision processes, and explicit feature. On the other hand, the disadvantages of the LBM are also kept by these models, such as limitation to uniform mesh, tie-up between the time step and the mesh spacing, and complex implementation of boundary conditions. To eliminate the drawbacks of the LBM, a lattice Boltzmann flux solver (LBFS) for single phase flows has been recently presented [31–33]. LBFS is a finite volume solver for direct update of the macroscopic flow variables at cell centers. The single-phase LB model is applied locally at the cell interface to reconstruct the viscous and inviscid fluxes simultaneously. Due to the local application of the LBM, LBFS not only successfully eliminates the previously mentioned drawbacks of the LBM but also effectively combines the advantages of the Navier–Stokes solvers and the LBM.

In this series of work, we aim to develop a multiphase lattice Boltzmann flux solver (MLBFS) for effective simulation of incompressible multiphase flows with both low and large density ratios. As mentioned above, the multiphase LB model is usually very complicated due to consideration of forcing terms in the model itself. This drawback can be overcome in the developed MLBFS since LB model is only applied at each cell interface locally. To be specific, we start from the macroscopic governing differential equations recovered by conventional multiphase LB models, which will be solved by the finite volume method. Then we consider the standard LBE without forcing terms and perform the Chapman–Enskog expansion analysis. From the analysis, we can establish relationships between fluxes in the governing differential equations and density distribution functions in the standard LBE. These relationships will be used to evaluate fluxes at each cell interface during finite volume discretization of governing differential equations. The forcing terms in the governing differential equations are directly treated by the finite volume method. Since the standard LB model is applied locally at each interface, and flow variables at cell centers are given directly from the solution of macroscopic governing equations, the physical boundary conditions can be directly implemented. Apart from the flow field, the evolution of the interface is modeled by the Cahn–Hilliard equation, which is solved by the fifth-order upwind scheme. Overall, the MLBFS can be effectively applied for multiphase flows with large density ratios on non-uniform grids, and retains the simplicity and advantages of the standard LBM. In addition, as compared with the conventional N–S solver, the present method is also much simpler and more flexible for applications on body-fitted grids by avoiding the application of staggered grid and the pressure–velocity coupling. The reliability and capability of the MLBFS will be validated by its application to simulate several test examples such as the two-phase co-current flows with density ratio up to 1000, Taylor–Couette flows in an annulus, droplet spreading on a flat plate, Rayleigh–Taylor instability and droplet splashing on a thin film.

2. Methodology

2.1. Governing equations

To show the governing equations of the MLBFS, the macroscopic Navier–Stokes (N–S) equations, recovered by the multiphase LB model [21], are described first. After that, the standard LBE model, which does not involve external forcing terms, will be analyzed by the multi-scale Chapman–Enskog expansion analysis. From the analysis, relationships between the fluxes in the governing differential equations and density distribution function in the standard LBE will be established. These relationships will then be incorporated into the final governing equations.