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A boundary integral equation domain decomposition method for electromagnetic scattering from large and deep cavities

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ABSTRACT

Electromagnetic scattering analysis of large and deep cavities embedded in an arbitrarily shaped host body is of high interest to the engineering community. The objective of this work is to investigate an effective boundary integral equation domain decomposition method for solving the cavity scattering problems. The key features of the proposed work include: (i) the introduction of individual electric and magnetic traces as unknowns for each sub-region, (ii) the development of a multi-trace combined field integral equation formulation for decomposed boundary value problem, and (iii) the derivation of optimized multiplicative Schwarz preconditioning using complete second order transmission condition. The proposed method can be viewed as an effective preconditioning scheme for the integral equation based solution of the cavity scattering problems. The strength and flexibility of the proposed method will be illustrated by means of several representative numerical examples.

1. Introduction

Accurate numerical analysis of electromagnetic (EM) scattering by large and deep cavities embedded in an arbitrarily shaped host body is an important problem with various applications. For example, a practical application of cavity scattering problem is shown in Fig. 1(a). Examples of cavities in the aircraft include air intakes of aircraft, jet engine inlet ducts, burner and combustion chambers. Of particular interest is the characterization and prediction of the radar cross section (RCS) of these cavities, because the scattering from the interior cavities contributes significantly to the overall RCS of a jet aircraft. Nonetheless, the computation of EM scattering from 3-D large and complex cavities does pose non-trivial challenges. We consider a plane wave scattering from a mockup elliptical jet engine inlet at Ku-band, as shown in Fig. 1(b). The inlet is large (1.26 m $\times$ 0.45 m opening), deep (5.02 m) and contains complex internal structure. Moreover, the interior surface of the inlet is coated with thin, high density and lossy coatings (uniform thickness of 1 mm). Clearly, it is a challenging large and multi-scale EM scattering problem.

There is a large literature available on computation and analysis of cavity scattering problems. Asymptotic techniques have been employed to tackle this problem, including the geometrical shooting and bouncing ray methods [1,2], and hybrid asymptotic high frequency and modal methods [3]. The applications of these methods are often limited to large and simple cavities. Another well-known technique is the differential equation method, which directly solves the Maxwell's equations in their differential form. Examples are the finite-difference time-domain methods [4], the finite element method [5–7], and

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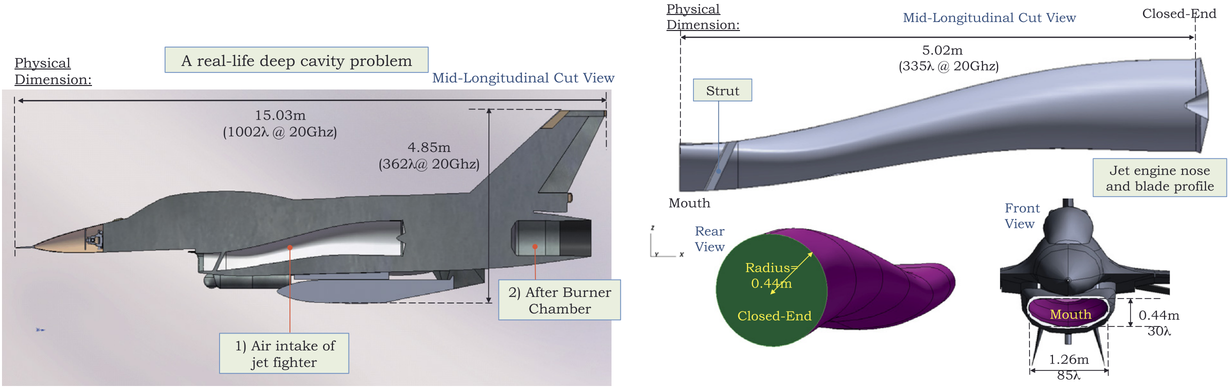

(a) A practical application of cavity scattering problem

(b) A mockup elliptical jet engine inlet at Ku-band

Fig. 1. Electromagnetic scattering from large and deep inlets embedded in an arbitrarily shaped host body.

hybrid finite element and boundary integral equation method [8,9]. One disadvantage of differential equation solvers is the numerical grid dispersion error. It results in an error in the phase velocity of wave and becomes significant after wave propagating over a large region. Hence, the grid discretization density has to increase with increased problem size, making these methods less attractive for electrically large cavity scattering problems. Boundary integral equation (BIE) method provides an appealing alternative [10–12]. Since both the analysis and unknowns reside only on the boundary surfaces of the objects, it often requires fewer unknowns to solve compared to a differential equation solver, where the unknowns scale volumetrically. More significantly, it is immune from the grid dispersion error and also very suitable for high density materials. However, applications of the BIE methods for the long/deep cavity scattering problems often lead to ill-conditioned matrix equations, and hence, cause slow convergence in iterative matrix solutions. The essential source of the ill-conditioning can be attributed to the physics of resonance in the internal cavity. It produces standing wave phenomena, which results in long-range strong coupling effects. The BIE methods with conventional near-field preconditioner are insufficient to address the long-range coupling effects effectively. This leads to poor convergence when using iterative matrix solvers. Clearly, we shall need an effective and reliable preconditioning strategy to assure fast convergence in iterative matrix solutions.

This work investigates the use of domain decomposition methods for the integral equation based solution of the cavity scattering problem. Domain decomposition (DD) methods have attracted considerable attention for solving partial differential equations. Many references are available in the literature [13–17]. Recently, they have also been extended to BIE methods [18–20]. Of particular interest is the boundary element tearing and interconnecting (BETI) method [21,22], where local boundary value problems are coupled through Lagrange multipliers. This method has been applied to solve electrostatic and elastostatic problems [23] and acoustic wave problems [24,25]. Moreover, there two recent works worth mentioning. One is the so-called multi-trace BIE formulations [26] for second-order transmission problems. These formulations have been developed and analyzed for acoustic [27] and EM [28,29] scattering at composite objects. The primary motivation of these methods was to find first-kind BIE formulations for composite objects that are amenable to operator preconditioning. In computational electromagnetics, the boundary integral equation DD method (BIE-DDM) has been introduced for large multi-scale EM simulations. Applications of the BIE-DDM for EM scattering from non-penetrable targets are presented in [30,31]. The treatment of general penetrable composite targets is discussed in [32,31]. Although it was driven by different objectives, the method also employs multi-trace spaces at the sub-region surface. A discussion and comparison of these two classes of formulations can be achieved in [33].

In this paper, we extend and consolidate the BIE-DDM [31] for solving the cavity scattering problem. The proposed method employs a hierarchical domain partitioning strategy. The first level of partitioning is based on the physical characteristics. An artificial surface is placed over the opening of the cavity as a transmission interface to decompose the computational domain into the interior bounded cavity region and the exterior host body region. Accordingly, the exterior traveling wave physics and interior resonance physics can be separated. Next, owing to potentially complex geometrical features inside the cavity, we further partition the interior cavity region into moderate-sized sub-regions, again through artificial surfaces. After the decomposition, the computational domain is partitioned into non-overlapping sub-domains. A multi-trace combined field integral equation formulation, whose unknowns include surface traces of both electric and magnetic fields, is employed for individual sub-domains. Subsequently, the required couplings between the sub-domains are completely channeled through the artificial surface at the sub-domain interfaces, via the application of local transmission conditions. Thus, the proposed method can be viewed as a form of the local multi-trace BIE formulation proposed in [29, Section 6] and [34, Section 5]. A particularly novel aspect of this work is the development of the optimized second order transmission conditions to improve the convergence in the DD algorithm. Numerical results verify the analysis and demonstrate the capabilities of the proposed method.

The rest of this paper is planned as follows: the next Section 2 gives a concise survey of relevant function spaces and trace theorems, which are needed for the rest of the paper. Then we introduce the notations of the domain decomposition