Journal of Computational Physics 278 (2014) 117–132

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Automated divertor target design by adjoint shape sensitivity analysis and a one-shot method

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ARTICLE INFO

Article history:
Received 27 January 2014
Received in revised form 14 August 2014
Accepted 15 August 2014
Available online 22 August 2014

Keywords:
Divertor design
Edge plasma modeling
Shape optimization
Shape sensitivity analysis
Continuous adjoint method
One-shot method

ABSTRACT

As magnetic confinement fusion progresses towards the development of first reactor-scale devices, computational tokamak divertor design is a topic of high priority. Presently, edge plasma codes are used in a forward approach, where magnetic field and divertor geometry are manually adjusted to meet design requirements. Due to the complex edge plasma flows and large number of design variables, this method is computationally very demanding. On the other hand, efficient optimization-based design strategies have been developed in computational aerodynamics and fluid mechanics. Such an optimization approach to divertor target shape design is elaborated in the present paper. A general formulation of the design problems is given, and conditions characterizing the optimal designs are formulated. Using a continuous adjoint framework, design sensitivities can be computed at a cost of only two edge plasma simulations, independent of the number of design variables. Furthermore, by using a one-shot method the entire optimization problem can be solved at an equivalent cost of only a few forward simulations. The methodology is applied to target shape design for uniform power load, in simplified edge plasma geometry.

1. Introduction

Power and particle exhaust are key performance issues for next-step fusion reactors. Divertors have to be designed such that they could safely handle the large power loads. Specifically, their design needs to prevent from exceeding limits imposed by the divertor materials and the plasma facing component cooling in order to avoid excessive material erosion, surface melting and failure of the component as a whole due to beyond-design power fluxes. At the same time, sufficient particle throughput, in particular Helium pumping capacity, has to be ensured.

In the divertor design process, numerical simulations of the plasma edge are heavily used to assess divertor performance. Typically, plasma edge codes such as B2-Eirene [1] are used as analysis tools, for example in the design of the ITER divertor [2]. Unfortunately, due to the complex nature of the edge plasma flows and the large number of design variables, extended parametric studies with these edge codes are computationally very demanding, precluding investigating a wide range of divertor geometries and operational points.

In this paper, we elaborate in detail a novel, optimization-based approach to computational divertor design. By recasting the design problem as a mathematical optimization problem, a range of computational tools developed over the past

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http://dx.doi.org/10.1016/j.jcp.2014.08.023
0021-9991/

decades, mainly in computational aerodynamics [3,4], becomes accessible. There, the aim may be to design airfoils for a given pressure profile [5,6], for drag reduction or optimal lift-to-drag ratio [7], to increase the pressure gain in compressor stages, or to shape ducts for minimum pressure or viscous losses [8]. Very efficient optimization algorithms have been developed, which solve entire design problems at an equivalent cost of only a few flow simulations [9–11], and initial studies have shown they may be very powerful for computational divertor design as well [12]. Section 2 reviews the necessary ingredients in the mathematical formulation of the divertor design problem. An example of a design problem is given. Next, conditions characterizing the solution to the design problem are elaborated in Section 3. Finally, Section 4 presents the results of a case study. Divertor target shapes are designed for uniform power load, using a simplified geometrical representation of the edge plasma.

2. Problem formulation

2.1. Divertor design as a mathematical optimization problem

In an optimization framework, the aim is to minimize a cost functional $J(\Omega, \mathbf{q})$ subject to a number of constraints. The functional $J(\Omega, \mathbf{q})$ is a performance measure for the design, which depends both on the shape of the simulated domain Ω, and on the plasma state variables $\mathbf{q}$. Typically, functionals are considered which are integrals over the domain Ω or (a part of) its boundary $\Sigma \equiv \partial\Omega$, for example

$$J_1(\Omega, \mathbf{q}) = \int_{\Omega} f(\mathbf{q})\,\mathrm{d}\omega, \tag{1}$$

$$J_2(\Omega, \mathbf{q}) = \int_{\Sigma} g(\mathbf{q}, \boldsymbol{\nu})\,\mathrm{d}\sigma, \tag{2}$$

with f and g sufficiently smooth functions. $\boldsymbol{\nu}$ is the outward pointing unit normal. The vector of plasma state variables $\mathbf{q}$, i.e. the particle density, velocity and temperature, in turn depends on the shape of the domain. This is expressed by the plasma state equations $B(\Omega, \mathbf{q}) = 0$, which are typically a set of partial differential equations (PDEs) with corresponding boundary conditions. They could be the full set of Braginskii equations [13] augmented with a (kinetic) model for the neutrals, or a somewhat simplified model as used in the present work (see Section 2.2.2). In order to emphasize the complex dependence of the state equations on the domain, $B(\Omega, \mathbf{q})$ is written explicitly as field equations $B(\mathbf{q})$ with boundary conditions $B_S(\mathbf{q}, \boldsymbol{\nu})$:

$$0 = B(\Omega, \mathbf{q}) = \begin{cases} B(\mathbf{q}) & \text{in } \Omega, \\ B_S(\mathbf{q}, \boldsymbol{\nu}) & \text{on } \Sigma. \end{cases} \tag{3}$$

The dependence on Ω is thus in the underlying domain and boundary, but also in the possible dependence of the boundary conditions on the normal.

The optimal design is the one which minimizes the cost functional. Combining all elements, the divertor design problem is stated as the mathematical optimization problem

$$\min_{\Omega \in \mathcal{O}, \mathbf{q}} \quad J(\Omega, \mathbf{q}) \quad \text{subject to} \quad B(\Omega, \mathbf{q}) = 0. \tag{4}$$

The state equations appear as a constraint in the optimization problem. Assuming every domain Ω uniquely defines a corresponding plasma state $\mathbf{q} = \mathbf{q}(\Omega)$ through the solution of the state equations, the problem can be formulated equivalently in terms of the reduced cost functional $\hat{J}$ as

$$\min_{\Omega \in \mathcal{O}} \quad \hat{J}(\Omega) \equiv J\big(\Omega, \mathbf{q}(\Omega)\big). \tag{5}$$

By restricting Ω to a set of admissible domains $\mathcal{O}$, direct constraints on the design space can be taken into account.

2.2. Example problem: divertor target shape design for uniform power load

2.2.1. Cost functional and shape parametrization

In order to optimize the energy load of the divertor targets, a cost functional is proposed which aims at achieving a uniform target load Q_d,

$$J(\Omega, \mathbf{q}) = \frac{1}{2} \int_{\mathrm{t}} (Q_\mathrm{o} - Q_\mathrm{d})^2\,\mathrm{d}\sigma. \tag{6}$$

The integral is taken at the divertor targets, AB and CD in Fig. 1. In the figure, the edge plasma geometry is represented both in (a) a cylindrical coordinate system, and (b) in a magnetic field aligned poloidal–radial coordinate system. Symmetry in the toroidal direction is assumed. This cost functional measures the square of the difference between the actual energy