Journal of Computational Physics 278 (2014) 400–415

Contents lists available at ScienceDirect

Journal of Computational Physics

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A positivity-preserving semi-implicit discontinuous Galerkin scheme for solving extended magnetohydrodynamics equations

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ARTICLE INFO

Article history:
Received 26 February 2014
Received in revised form 20 August 2014
Accepted 26 August 2014
Available online 3 September 2014

Keywords:
Positivity-preserving
Discontinuous Galerkin
Relaxation method
Magnetohydrodynamics
Two-fluid
HED plasma

ABSTRACT

A positivity-preserving discontinuous Galerkin (DG) scheme [42] is used to solve the Extended Magnetohydrodynamics (XMHD) model, which is a two-fluid model expressed with a center-of-mass formulation. We prove that DG scheme with a positivity-preserving limiter is stable for the system governed by the XMHD model or the resistive MHD model. We use the relaxation system formulation [28] for describing the XMHD model, and solve the equations using a split level implicit–explicit time advance scheme, stepping over the time step constraint imposed by the stiff source terms. The magnetic field is represented in an exact locally divergence-free form of DG [23], which greatly improves the accuracy and stability of MHD simulations. As presently constructed, the method is able to handle a wide range of density variations, solve XMHD model on MHD time scales, and provide greatly improved accuracy over a Finite Volume implementation of the same model.
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1. Introduction

In High Energy Density (HED) plasma systems, we usually must deal with a wide dynamic range of current carrying densities. When density is so low that the scale lengths are comparable to ion inertial length, the single fluid model would break down. For these problems two-fluid physics is essential, but the applicability of existing numerical methods [6,16,17,19,25,34] in this plasma regime is still limited. In this paper we develop a discontinuous Galerkin (DG) method to solve an extended Magnetohydrodynamics (XMHD) model based on a relaxation formulation, and will demonstrate that this numerical scheme has high accuracy, is computationally efficient, and can handle the large dynamic density range for HED plasmas.

DG schemes are widely used in the hyperbolic algorithms community, since the method can handle complex geometries, has an arbitrary order of accuracy, and is efficient in parallel calculations [9–12,31]. In addition, DG has further advantages for HED plasmas, because the density range is very wide, one often faces a problem where the initial density profile is nearly a δ-singularity (e.g. a pinched-down wire). Such problems are difficult to approximate numerically, and most previous techniques are based on the modification of the singularities with smooth kernels in some narrow region (e.g. [37,40] and the references therein), and hence smear such singularities. However, DG methods depend on the weak form of the equations and can solve such problems without modification, leading to a very accurate result [38]. The main challenge

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http://dx.doi.org/10.1016/j.jcp.2014.08.044
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with DG implementations lies in preserving the stability of the system. Near discontinuities, strong oscillations often appear that might send the physically positive quantities negative. When this occurs, the numerical simulation may break down. In our system, the quantities that should be preserved positive are: density ρ, pressure P, energy $\mathcal{E}$. To ensure these quantities satisfy a positivity-preserving property, one could use a Total Variation Diminishing (TVD) scheme; however all TVD schemes will degenerate to lower order accuracy at smooth extrema [41]. In [42], a positivity-preserving high order DG scheme is developed for solving the compressible Euler equations, and in [39], the authors demonstrated the L^1-stability of such a scheme. This scheme has been applied on solving the ideal magnetohydrodynamic (MHD) equations [3,7]. In this paper, we will demonstrate that the positivity-preserving DG scheme can establish a positivity-preserving property for a system described by an XMHD model or a resistive MHD model, thus preserving the stability of the system.

For solving the XMHD model, we use the relaxation algorithm proposed in [28] to solve the combination of generalized Ohm's law (GOL) and the Maxwell–Ampére law for the electric field. The relaxation method is essentially a semi-implicit time differencing proven to converge to the solution of the algebraic equations that are the stiff source terms set equal to zero. In the case of XMHD this means the solution converges to GOL in which the electron inertial terms are neglected and to the Maxwell equation in which the displacement current term is neglected (Ampére's Law). Using this algorithm, one can avoid the constraint imposed by the under-resolved stiff source terms to step over electron plasma and cyclotron frequencies, which are often under-resolved in the characteristic regime, and thereby allow one to solve the XMHD equations on MHD time scales. In this algorithm, the Hall term is locally implicit avoiding the substantial effort in solving a large linear system. Furthermore, with XMHD model, in low-density region, the current is suppressed by both Hall and electron inertial terms, one is able to use the unmodified Spitzer resistivity, which makes the plasma–vacuum transition automatic and physical. This algorithm has been implemented into an XMHD code called PERSEUS with a second-order finite volume (FV) scheme. Because of the extended stencil, FV is often too diffusive to characterize small-scale fluid instabilities and to resolve the local details of a shock structure without a large number of cells. On the other hand, DG scheme is more compact in the sense that every cell is treated independently, and the solution is localized within a cell. This leads to a reduced numerical diffusion. In this paper, we compare the performance of a DG formulation XMHD code to that of an FV formulation, and will demonstrate through numerical tests that DG has a significant advantage over the same order of FV in both memory and CPU time. Additionally we compare selected results with those found from the MHD model computed using the same algorithm. This comparison demonstrates the viability of the method for solving XMHD problems as well as to point out important deficiencies in MHD.

This paper is organized as follows. In Section 2 we introduce the XMHD model and the relaxation algorithm. In Section 3, we construct the DG scheme used in this paper, and demonstrate that a positivity-preserving limiter can preserve the positivity-preserving property in a system governed by an XMHD model or a resistive MHD model, we will also give a brief introduction on the implementation of our algorithm. In Section 4, we present the results of numerical tests, in some of the tests, we do a comparison with FV, to show that DG is more accurate and more efficient than an FV implementation. We also give examples of the method applied to fundamental plasma physics problems and provide a comparison with the MHD model for the same problems. Concluding remarks are provided in Section 5.

2. XMHD model

2.1. Governing equations

In the study of HED plasmas, we are often in a regime where the ion inertial length or even the electron inertial length are resolved by the cell dimensions. When this occurs, standard MHD does not accurately describe the system, and a more generalized model is called for. The two-fluid model expressed with a center-of-mass formulation, which is the XMHD model, covers most of the physics occurring in this regime. The primary exception being due to space-charge effects, which is neglected in the quasineutral XMHD model. The XMHD model is given by:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{2.1}$$

$$\frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u} + \mathbf{I}P) = \mathbf{J} \times \mathbf{B} \tag{2.2}$$

$$\frac{\partial \mathcal{E}_n}{\partial t} + \nabla \cdot \left[\mathbf{u}(\mathcal{E}_n + P)\right] = \mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) + \eta \mathbf{J}^2 \tag{2.3}$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0 \tag{2.4}$$

$$\frac{\partial \mathbf{E}}{\partial t} - c^2 \nabla \times \mathbf{B} = -\frac{1}{\epsilon_0}\mathbf{J} \tag{2.5}$$

$$\frac{\partial \mathbf{J}}{\partial t} + \nabla \cdot \left(\mathbf{u}\mathbf{J} + \mathbf{J}\mathbf{u} - \frac{1}{n_e e}\mathbf{J}\mathbf{J} - \frac{e}{m_e}\mathbf{I}P_e\right) = \frac{n_e e^2}{m_e}\left(\mathbf{E} + \mathbf{u} \times \mathbf{B} - \eta \mathbf{J} - \frac{1}{n_e e}\mathbf{J} \times \mathbf{B}\right) \tag{2.6}$$

$$\partial_t S_e + \nabla \cdot (\mathbf{u}_e S_e) = (\gamma - 1) n_e^{1-\gamma} \eta \mathbf{J}^2. \tag{2.7}$$

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