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Wave simulation in 2D heterogeneous transversely isotropic porous media with fractional attenuation: A Cartesian grid approach

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ABSTRACT

A time-domain numerical modeling of transversely isotropic Biot poroelastic waves is proposed in two dimensions. The viscous dissipation occurring in the pores is described using the dynamic permeability model developed by Johnson–Koplik–Dashen (JKD). Some of the coefficients in the Biot-JKD model are proportional to the square root of the frequency. In the time-domain, these coefficients introduce shifted fractional derivatives of order 1/2, involving a convolution product. Based on a diffusive representation, the convolution kernel is replaced by a finite number of memory variables that satisfy local-in-time ordinary differential equations, resulting in the Biot-DA (diffusive approximation) model. The properties of both the Biot-JKD and the Biot-DA models are analyzed: hyperbolicity, decrease of energy, dispersion. To determine the coefficients of the diffusive approximation, two approaches are analyzed: Gaussian quadratures and optimization methods in the frequency range of interest. The nonlinear optimization is shown to be the better way of determination. A splitting strategy is then applied to approximate numerically the Biot-DA equations. The propagative part is discretized using a fourth-order ADER scheme on a Cartesian grid, whereas the diffusive part is solved exactly. An immersed interface method is implemented to take into account heterogeneous media on a Cartesian grid and to discretize the jump conditions at interfaces. Numerical experiments are presented. Comparisons with analytical solutions show the efficiency and the accuracy of the approach, and some numerical experiments are performed to investigate wave phenomena in complex media, such as multiple scattering across a set of random scatterers.

1. Introduction

A porous medium consists of a solid matrix saturated with a fluid that circulates freely through the pores [1–3]. Such media are involved in many applications, modeling for instance natural rocks, engineering composites [4] and biological materials [5]. The most widely used model describing the propagation of mechanical waves in porous media has been proposed by Biot in 1956 [1,6]. It includes two classical waves (one "fast" compressional wave and one shear wave), in addition

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to a second "slow" compressional wave, which is highly dependent on the saturating fluid. This slow wave was observed experimentally in 1980 [7], thus confirming the validity of Biot's theory.

Two frequency regimes have to be distinguished when dealing with poroelastic waves. In the low-frequency range (LF), the flow inside the pores is of Poiseuille type [1]. The viscous effects are then proportional to the relative velocity of the motion between the fluid and the solid components. In the high-frequency range (HF), modeling the dissipation is a more delicate task. Biot first presented an expression for particular pore geometries [6]. In 1987, Johnson–Koplik–Dashen (JKD) published a general expression for the HF dissipation in the case of random pores [8], where the viscous efforts depend on the square root of the frequency. No particular difficulties are raised by the HF regime if the solution is computed in the space-frequency domain [9,10]. On the contrary, the computation of HF waves in the space–time domain is much more challenging. Time fractional derivatives are then introduced, involving convolution products [11]. The past of the solution must be stored, which dramatically increases the computational cost of the simulations.

The present work proposes an efficient numerical model to simulate the transient poroelastic waves in the full frequency range of Biot's model. In the high-frequency range, only two numerical approaches have been proposed in the literature to integrate the Biot-JKD equations directly in the time-domain. The first approach consists in a straightforward discretization of the fractional derivatives defined by a convolution product in time [12]. In the example given in [12], the solution is stored over 20 time steps. The second approach is based on the diffusive representation of the fractional derivative [13]. The convolution product is replaced by a continuum of memory variables satisfying local differential equations [14]. This continuum is then discretized using Gaussian quadrature formulae [15–17], resulting in the Biot-DA (diffusive approximation) model. In the example proposed in [13], 25 memory variables are used, which is equivalent, in terms of memory requirement, to storing 25 time steps. The idea of using memory variables to avoid convolution products is close to the strategy commonly used in viscoelasticity [18].

The concern of realism leads us also to tackle with anisotropic porous media. Transverse isotropy is commonly used in practice. It is often induced by Backus averaging, which replaces isotropic layers much thinner than the wavelength by a homogeneous isotropic transverse medium [19]. To our knowledge, the earliest numerical work combining low-frequency Biot's model and transverse isotropy is based on an operator splitting in conjunction with a Fourier pseudospectral method [20]. Recently, a Cartesian-grid finite volume method has been developed [21]. One of the first works combining anisotropic media and high-frequency range is proposed in [22]. However, the diffusive approximation proposed in the latter article has three limitations. Firstly, the quadrature formulae make the convergence towards the original fractional operator very slow. Secondly, in the case of low frequencies, the Biot-DA model does not converge towards the Biot-LF model. Lastly, the number of memory variables required for a given accuracy is not specified.

The present work extends and improves our previous contributions about the modeling of poroelastic waves. In [23], we addressed 1D equations in the low-frequency range, introducing a splitting of the PDE. 2D generalizations for isotropic media required to implement space–time mesh refinement [24,25]. Diffusive approximation of the fractional derivatives in the high-frequency range was introduced in [26] and generalized in 2D in [27]. Compared with [27], the originality of the present paper is threefold:

1. incorporation of anisotropy. The numerical scheme and the discretization of the interfaces need to be largely modified accordingly;
2. new procedure to determine the coefficients of the diffusive approximation. In [26,27], we used a classical least-squares optimization. It is much more accurate than the Gauss–Laguerre technique proposed in [13]. But in counterpart, some coefficients are negative, which prevents to conclude about the well-posedness of the diffusive model. Here, we fix this problem by using optimization with constraint of positivity, based on Shor's algorithm. Moreover, the accuracy of this new method is largely improved compared with the linear optimization;
3. theoretical analysis. A new result about the eigenvalues of the diffusion matrix is introduced and the energy analysis is extended to anisotropy.

This article is organized as follows. The original Biot-JKD model is outlined in Section 2 and the diffusive representation of fractional derivatives is described. The energy decrease is proven, and a dispersion analysis is done. In Section 3, an approximation of the diffusive model is presented, leading to the Biot-DA system. The properties of this system are also analyzed: energy, hyperbolicity and dispersion. Determination of the quadrature coefficients involved in the Biot-DA model are investigated in Section 3.4. Gaussian quadrature formulae and optimization methods are successively proposed and compared, the latter being finally preferred. The numerical modeling of the Biot-DA system is addressed in Section 4, where the equations of evolution are split into two parts: the propagative part is discretized using a fourth-order finite-difference scheme, and the diffusive part is solved exactly. An immersed interface method is implemented to account for the jump conditions and for the geometry of the interfaces on a Cartesian grid when dealing with heterogeneous media. Numerous numerical experiments are presented in Section 5, validating the method developed in this paper. In Section 6, a conclusion is drawn and some future lines of research are suggested.