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# Cross-code comparisons of mixing during the implosion of dense cylindrical and spherical shells

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ABSTRACT

We present simulations of the implosion of a dense shell in two-dimensional (2D) spherical and cylindrical geometry performed with four different compressible, Eulerian codes: `RAGE`, `FLASH`, `CASTRO`, and `PPM`. We follow the growth of instabilities on the inner face of the dense shell. Three codes employed Cartesian grid geometry, and one (`FLASH`) employed polar grid geometry. While the codes are similar, they employ different advection algorithms, limiters, adaptive mesh refinement (AMR) schemes, and interface-preservation techniques. We find that the growth rate of the instability is largely insensitive to the choice of grid geometry or other implementation details specific to an individual code, provided the grid resolution is sufficiently fine. Overall, all simulations from different codes compare very well on the fine grids for which we tested them, though they show slight differences in small-scale mixing. Simulations produced by codes that explicitly limit numerical diffusion show a smaller amount of small-scale mixing than codes that do not. This difference is most prominent for low-mode perturbations where little instability finger interaction takes place, and less prominent for high- or multi-mode simulations where a great deal of interaction takes place, though it is still present. We present `RAGE` and `FLASH` simulations to quantify the initial perturbation amplitude to wavelength ratio at which metrics of mixing agree across codes, and find that bubble/spike amplitudes are converged for low-mode and high-mode simulations in which the perturbation amplitude is more than 1% and 5% of the wavelength of the perturbation, respectively. Other metrics of small-scale mixing depend on details of multi-fluid advection and do not converge between codes for the resolutions that were accessible.

## 1. Introduction

In a problem involving the implosion of a sphere or cylinder, Rayleigh–Taylor and Richtmyer–Meshkov instabilities may both be present. The Rayleigh–Taylor (RT) instability develops when the density and pressure gradients in a fluid are op-

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posed, as when a denser fluid rests atop a lighter fluid in a gravitational field, or when a lighter fluid accelerates a denser fluid. The Richtmyer–Meshkov (RM) instability develops when a shock passes through an interface between a heavy and a light fluid.

Most simulations and physical experiments to study the details of the growth of these instabilities have been performed in planar geometry [1,2]. However, Rayleigh–Taylor and Richtmyer–Meshkov instabilities also play important roles in non-planar problems. Instability growth on such disparate scales as inertial confinement fusion (ICF) capsules and the interiors of pre-supernova stars may dramatically influence the evolution of these implosions [3,4].

Simulations of instabilities in radially-converging flows present extra challenges beyond those present in simulations of instabilities in planar interfaces; in particular, the ideal spatial coordinate system and grid geometry are no longer obvious. While a spherical grid geometry may seem appealing for following the flow of fluid in spherical implosions, the simplicity of mesh-aligned flow is lost once turbulence develops. Spherical grid geometry also introduces coordinate singularities that impose a preferred orientation on the simulation, and features may develop differently along the axis than at other points around the sphere. In multi-physics problems, such features can expand and contaminate regions far from the axis. Cartesian grid geometries avoid any coordinate singularities and are suitable for simulating a wide variety of physical problems, but using them to simulate spherical implosions raises questions about the spatial resolution required not just for simulating curved interfaces on rectilinear meshes but also for following the growth of perturbations imposed on those interfaces.

Youngs and Williams [5] used a Lagrange-remap code on a 3D spherical polar mesh to simulate turbulent mixing in a sector of a spherical implosion with random perturbations applied initially to the interface between light and dense fluids. The authors found that the width of the mixing zone shrank slightly as they increased the resolution of the mesh but that, at the finest mesh resolution they used, the mixing zone width seemed to have converged.

Do simulations in other coordinate systems, on other grids, achieve similar levels of convergence? To what degree do measurements of the mixing zone width in spherical implosions agree between Cartesian and curvilinear meshes? Do other diagnostics of mixing and turbulence demonstrate the same degree of convergence as mesh resolution increases?

In this paper, we investigate how the choice of grid geometry influences simulations of radially-converging flows by comparing simulations from four codes: `RAGE` [6], `CASTRO` [7], `FLASH` [8–10], and `PPM` [11]. All four codes model compressible hydrodynamics on Eulerian meshes, but they differ in the details of their discretization schemes, their shock-capturing methods, limiters, and steepeners, their treatment of materials in mixed cells, and their support for different coordinate systems. We assess the impact of the choice of grid geometry on the ability of `RAGE` and `CASTRO` to follow the growth of turbulence from perturbed interfaces, and we examine what is required to follow perturbations with different wavelengths and initial amplitudes. We find that perturbation growth is not dependent on the choice of grid geometry, but is influenced by the numerical choices employed in different codes.

Simulations of turbulence in converging flows ultimately require fully three-dimensional spatial meshes [12]. However, comparisons of results between codes can take place using any spatial mesh, and lessons about the advantages or disadvantages of different grid geometries, or the spatial resolution required to reliably measure various diagnostics of mixing and turbulence, should apply to both 2D and 3D simulations. Because 3D turbulence is fundamentally different from 2D mixing, some conclusions drawn from the 2D study may need to be modified for 3D applications. To obtain the most benefit with the least computational cost, we focus in this paper on 2D simulations of cylindrical and spherical implosions.

We examine simulations to which single long-wavelength, single short-wavelength, and multi-mode perturbations were applied. The long-wavelength perturbations give us the largest separation of scales between those we introduce explicitly and those introduced numerically from the mesh, while the short-wavelength perturbations provide us with a way to check the variation of perturbation growth with azimuth as well as a test of the regime in which individual unstable features interact extensively with each other. The multi-mode simulations test both of these regimes simultaneously.

For simulations of cylindrical implosions (where the two-dimensional spatial domain is a slice perpendicular to the axis of the cylinder), we compare results from `RAGE` and `PPM` using Cartesian meshes with results from `FLASH` using a polar mesh. `FLASH`'s polar mesh allows us to simulate an unperturbed converging cylindrical interface; comparing with simulations of unperturbed interfaces on Cartesian meshes exposes the perturbations that arise specifically from the use of a Cartesian mesh, as the curved interface crosses mesh cell boundaries around the cylinder. Once we introduce explicit initial perturbations, the three codes produce similar results; the nonlinear growth of the instabilities ensures that, for sufficiently large initial perturbation amplitude, the imposed perturbations grow faster than any perturbations introduced by the mesh.

For simulations of spherical implosions, we compare results from `RAGE`, `CASTRO`, and `FLASH` using 2D $r$–$z$ meshes. As in the simulations of cylindrical implosions, we expect there to be an initial perturbation amplitude above which our imposed perturbations grow faster than perturbations arising from the mesh, and we expect that this amplitude will have some dependence on the wavelength of the perturbation and the resolution of the mesh. We vary the amplitude of the perturbation for a fixed problem and fixed maximum resolution to find the amplitude (at a given perturbation wavelength) at which the growth of the imposed perturbation dominates over mesh-induced features.

We describe the initial fluid configuration, the method for driving the implosion, the imposed perturbations, and the mesh resolutions used in the simulations in Section 2. We review the capabilities and algorithmic features of the four codes in Section 3. We compare the results of the simulations, including a variety of diagnostics of mixing and instability growth, in Section 4. We conclude, in Section 5, with a discussion of requirements for simulating perturbed interfaces in radially-converging flows.