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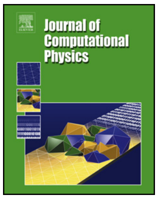

A robust method for handling low density regions in hybrid simulations for collisionless plasmas

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ABSTRACT

A robust method to handle vacuum and near vacuum regions in hybrid simulations for space and astrophysical plasmas is presented. The conventional hybrid simulation model dealing with kinetic ions and a massless charge-neutralizing electron fluid is known to be susceptible to numerical instability due to divergence of the whistler-mode wave dispersion, as well as division-by-density operation in regions of low density. Consequently, a pure vacuum region is not allowed to exist in the simulation domain unless some ad hoc technique is used. To resolve this difficulty, an alternative way to introduce finite electron inertia effect is proposed. Contrary to the conventional method, the proposed one introduces a correction to the electric field rather than the magnetic field. It is shown that the generalized Ohm's law correctly reduces to Laplace's equation in a vacuum which therefore does not involve any numerical problems. In addition, a variable ion-to-electron mass ratio is introduced to reduce the phase velocity of high frequency whistler waves at low density regions so that the stability condition is always satisfied. It is demonstrated that the proposed model is able to handle near vacuum regions generated as a result of nonlinear self-consistent development of the system, as well as pure vacuum regions set up at the initial condition, without losing the advantages of the standard hybrid code.

1. Introduction

Numerical simulations have been an essential tool to investigate complicated nonlinear phenomena occurring in space and astrophysical plasmas. Although the conventional magnetohydrodynamics (MHD) proves itself useful to describe macroscopic plasma dynamics even in the collisionless regime in which the mean free path for Coulomb collisions is comparable to or larger than the system size, it does not necessarily mean that one can completely ignore important kinetic physics. For example, it is well recognized that one must take into account kinetic effect to understand magnetic reconnection, which has been one of the key processes in magnetospheric physics affecting plasma transport, driving global convection, and perhaps triggering substorms. It is now becoming more and more popular to consider that magnetic reconnection plays a key role in astrophysical environments as well. Another example in which kinetic effect is central is the problem of particle acceleration in collisionless shocks. It requires seamless treatment of both microscopic and macroscopic physics because small-scale phenomena primarily determine the acceleration of low energy particles (or "injection"), while the transport of higher energy particles is predominantly governed by characteristics of MHD turbulence. Kinetic numerical simulations that can simultaneously deal with both macroscopic and microscopic dynamics of the collisionless plasma are indeed essential to investigate

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these important issues. Among those proposed so far, the best numerical technique for this purpose is probably the hybrid simulation, in which ions are treated kinetically whereas electrons are assumed to be a massless charge-neutralizing fluid [23,15].

The concept of the hybrid simulation is indeed promising in that it enables us to access the ion dynamics, while seemingly less important but more computationally demanding electron physics has been factored out. It has been widely used to study elementary processes such as plasma instabilities, magnetic reconnection, collisionless fast and slow shocks (e.g., [14,22,20,16,8]). With rapidly increasing computational resources, one may now be able to use a simulation box which is large enough to include the global scale as well. Recently, attempts have been made to model the interaction between the solar wind and relatively small unmagnetized and magnetized solar system bodies by using global hybrid simulations (e.g., [19,12,21,10,2]). On the other hand, it has been well known that hybrid simulations are in practice susceptible to numerical instability. Despite the long history of this technique, to the authors knowledge, any fundamental solutions to this problem has not been given. It is indeed a serious obstacle that hinders application to many important and interesting problems in space and astrophysical plasma physics. The primary purpose of the present paper is to provide a practical solution to the problem of numerical stability in the hybrid simulation. As we will see below, this can be realized by introducing a new way to include finite electron inertia effect.

It is well known that the Alfvén wave at short wavelength comparable to ion inertia length has dispersion due to the decoupling between ion and electron dynamics. There thus appears the whistler mode whose frequency diverges as $\omega \propto k^2$. This means that the maximum phase velocity in the system increases rapidly without bound, implying numerical difficulty. This is probably a part of the reasons for the numerical instability in hybrid simulations. It is thus easy to expect that inclusion of finite electron inertia can help stabilizing the simulation because the maximum phase velocity in this case is limited by roughly the electron Alfvén speed. Even with finite electron inertia, however, a numerical problem arises in regions of low density. This is obviously due to the division-by-density operation needed to calculate the electric field from ion moment quantities, which makes it impossible to handle such (near) vacuum regions. In practice, numerical difficulty arises even long before this limit is reached because the Alfvén speed increases as the density decreases, imposing a severe restriction on the simulation time step.

The method we propose in the present paper essentially resolves all these numerical difficulties. Our strategy is also to introduce finite electron inertia effect to limit the maximum phase velocity in the system. We argue that the way in which the electron inertia is introduced is a key to solve the problem. An electron inertia correction term has conventionally been introduced to the magnetic field and its electric field counterpart is often neglected (e.g., [13,18,17]). By modifying the procedure so that the correction is introduced directly to the electric field, we show that the division-by-density operation is almost eliminated from the simulation procedure. In addition to this, to reduce the maximum wave phase velocity in a low density region, the ion-to-electron mass ratio is considered to be a variable quantity. That is, the mass ratio is reduced locally so that the CFL (Courant–Friedrichs–Lewy) condition is automatically satisfied. We demonstrate that the proposed model implemented in a one-dimensional (1D) hybrid simulation code can successfully follow nonlinear evolution of the system even when extremely low density regions appear as a result of strong instabilities. Furthermore, we also show that the code is able to handle pure vacuum regions, as well as the interface between vacuum and finite density plasma regions. These features suggest that the present model is indeed very robust and will help stabilizing simulations applied to many important problems in space and astrophysical plasmas.

The present paper is organized as follows. First, we present a simulation model in Section 2, in which a new way to introduce finite electron inertia is discussed. Numerical implementation is explained in Section 3. Section 4 shows simulation results of several test problems. Finally, summary and conclusions are given in Section 5.

2. Simulation model

2.1. Standard hybrid model

For the sake of completeness and to clarify the differences, we first describe the standard hybrid model. Readers who are already familiar with the hybrid model and its assumptions can skip this subsection. Tutorials and comprehensive reviews of the hybrid code are found elsewhere [23,15].

The basic equations used in the hybrid model are consisting of equation of motion for individual ions and for a fluid electrons

$$\frac{d\mathbf{x}_j}{dt} = \mathbf{v}_j, \tag{1}$$

$$\frac{d\mathbf{v}_j}{dt} = \frac{q_j}{m_j}\left(\mathbf{E} + \frac{\mathbf{v}_j}{c} \times \mathbf{B}\right), \tag{2}$$

$$\frac{d\mathbf{v}_e}{dt} = -\frac{e}{m_e}\left(\mathbf{E} + \frac{\mathbf{v}_e}{c} \times \mathbf{B}\right) - \frac{1}{n_e m_e}\nabla \cdot \mathbf{P}_e, \tag{3}$$

where the subscript j and e indicate the indices for individual ions and the electron fluid and other notations are standard.