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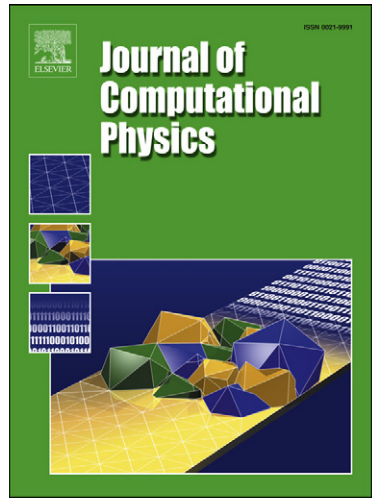

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A new Front Tracking Scheme for the Ultra-Relativistic Euler Equations

Mahmoud A.E. Abdelrahman[1] * and Matthias Kunik[2] †

[1] *Department of Mathematics, Faculty of Science, Mansoura University, 35516 Mansoura, Egypt*

[2] *Institute for Analysis and Numerics, Otto-von-Guericke University Magdeburg, Universitätsplatz 2, D-39106 Magdeburg, Germany*

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Abstract The ultra-relativistic Euler equations for an ideal gas are described in terms of the pressure p, the spatial part $\mathbf{u} \in \mathbb{R}^3$ of the dimensionless four-velocity and the particle density n. Two schemes for these equations are presented in one space dimension, namely a front tracking and a cone-grid scheme. A new front tracking technique for the ultra-relativistic Euler equations is introduced, which gives weak solutions. The front tracking method is based on piecewise constant approximations to Riemann solutions, called front tracking Riemann solutions, where continuous rarefaction waves are approximated by finite collections of discontinuities, so called non-entropy shocks. This method can be used for analytical as well as for numerical purposes. A new unconditionally stable cone-grid scheme is also derived in this paper, which is based on the Riemann solution for the ultra-relativistic Euler equations. Both schemes are compared by two numerical examples, where explicit solutions are known.

Keywords: Relativistic Euler equations, conservation laws, hyperbolic systems, Lorentz transformations, shock waves, entropy conditions, rarefaction waves, non-entropy shocks, front tracking scheme.

AMS subject classifications: 35L45, 35L60, 35L65, 35L67, 76Y05, 65M99

1 Introduction

In this paper we are concerned with the ultra-relativistic equations for a perfect fluid in Minkowski space-time, which can be written in the following form by using Einstein's summation convention in [21]:

$$\frac{\partial T^{\alpha\beta}}{\partial x^{\beta}} = 0, \quad \frac{\partial N^{\alpha}}{\partial x^{\alpha}} = 0, \tag{1.1}$$

where

$$T^{\alpha\beta} = -pg^{\alpha\beta} + 4pu^{\alpha}u^{\beta} \tag{1.2}$$

denotes the energy-momentum tensor for the ideal ultra-relativistic gas. Here p represents the pressure, $\underline{u} \in \mathbb{R}^3$ is the spatial part of the four-velocity $(u^0, u^1, u^2, u^3) = (\sqrt{1 + |\underline{u}|^2}, \underline{u})$ and $g^{\alpha\beta}$ denotes the flat Minkowski metric, which is

$$g^{\alpha\beta} = \begin{cases} +1, & \alpha = \beta = 0\,, \\ -1, & \alpha = \beta = 1,2,3\,, \\ 0, & \alpha \neq \beta\,, \end{cases} \tag{1.3}$$

*mahmoud.abdelrahman@mans.edu.eg; mahmoud.abdelrahman@st.ovgu.de; Tel +2 050 2242388; Fax +20 050 2246781
†matthias.kunik@ovgu.de; Tel +49 391 6712877; Fax +49 391 6718073