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A hybrid transport-diffusion model for radiative transfer in absorbing and scattering media

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ABSTRACT

A new multi-scale hybrid transport-diffusion model for radiative transfer is proposed in order to improve the efficiency of the calculations close to the diffusive regime, in absorbing and strongly scattering media. In this model, the radiative intensity is decomposed into a macroscopic component calculated by the diffusion equation, and a mesoscopic component. The transport equation for the mesoscopic component allows to correct the estimation of the diffusion equation, and then to obtain the solution of the linear radiative transfer equation. In this work, results are presented for stationary and transient radiative transfer cases, in examples which concern solar concentrated and optical tomography applications. The Monte Carlo and the discrete-ordinate methods are used to solve the mesoscopic equation. It is shown that the multi-scale model allows to improve the efficiency of the calculations when the medium is close to the diffusive regime. The proposed model is a good alternative for radiative transfer at the intermediate regime where the macroscopic diffusion equation is not accurate enough and the radiative transfer equation requires too much computational effort.

1. Introduction

Linear transport models are applied in many research fields including neutron transport [1], thermal radiation [2], medical imaging [3] or atmospheric physics [4], among others. In this work, we focus on radiative transfer models in absorbing and scattering media for applications such as solar energy processes and optical tomography. Two principal models are generally used for linear radiation transport: the radiative transfer equation (RTE) defined at the kinetic scale and the macroscopic diffusion equation (DE).

The RTE is a mesoscopic model which offers a better accuracy than the DE but requires a higher computational effort. Several numerical methods such as the Monte Carlo method (MCM), the discrete-ordinates method (DOM) or the finite-volume method (FVM) have been developed over the last decades in order to solve the RTE with precision and efficiency. The DOM and the FVM are deterministic methods, frequently applied in engineering problems, which are generally faster than the MCM [2]. However, with the increase of computational power, the MCM becomes more attractive and has been

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used in various engineering applications [5]. Accordingly, the Monte Carlo algorithms have been significantly improved in concentrated solar processes [6], remote sensing [7] or for sensitivity estimations [8].

All these methods based on the RTE converge slowly when the medium is close to the diffusive regime. Accordingly, the computational requirements increase significantly with the scattering optical thickness. In the MCM, the CPU time increases with the number of times that photons scatter along their optical path, and the statistical error increases as well. In the DOM or FVM, the numerical parameters must respect strong constraints for stability reasons, e.g., the size of the control volumes should be smaller than the mean free path which becomes very small if the scattering optical thickness is large. At the diffusive regime, the DE is generally used if it is assumed that the radiative intensity is almost isotropic (P1 approximation), and that the radiative heat flux scales with the gradient of the incident radiation [2]. The DE is much easier to solve than the RTE, and is therefore an efficient model in highly scattering media. But the DE has some major drawbacks in the proximity of boundaries or sources, or in regions where the medium is not optically thick, where the P1 approximation is not valid anymore.

The objective of this work is to propose a radiation model for multi-scale problems where both diffusive and kinetic regimes are present. We will focus firstly on a case related to radiation transport in a volumetric receiver in concentrated solar applications. High temperature volumetric solar receivers have been a research topic for more than 30 years [9]. The aim of volumetric receivers is to use semi-transparent media (particles, wire meshes, fibers, honeycombs, reticulated porous ceramics) to absorb solar radiation deep inside the absorber in order to decrease losses by reflections and thermal emission. Recent studies [10,11] used the DE for the prediction of thermal radiation inside a volumetric solar absorber. The DE leads to acceptable computation time, but it cannot estimate accurately the losses at the inlet boundary. In these applications, a radiation multi-scale model, which is able to predict correctly the losses by reflections or emission close to the surface irradiated by the concentrated solar fluxes, and to predict correctly radiative transfer inside the volumetric absorber where the medium is close to the diffusive regime, is needed.

Another example studied in this work is related to optical tomography applications. The propagation of a short-pulse laser into a biological tissue is used to estimate the optical properties of the tissue and to detect inhomogeneities and tumors [12,13]. In biological tissue, the medium is generally optically thick, and scattering dominates over absorption, which means that the DE can be assumed for radiation model [14]. However, the diffusion approximation has some limitations at the boundaries of the system where the short-pulse laser is emitted. The collimated irradiation cannot be simulated accurately with the DE, and needs a kinetic description. Moreover, scattering is typically forward-peaked in tissues [15], which enhances the need for a mesoscopic model close to the boundaries. In such cases, a transient multi-scale radiation model is needed to simulate the propagation of the short-pulse laser inside the tissue.

In multi-scale problems, a solution is to couple the DE with the RTE using a domain decomposition method, in which the system is decomposed into a mesoscopic subdomain where the RTE is solved and a macroscopic subdomain where the DE is solved. This approach has been tested in various studies [15–18]. In these works, the treatment of the geometric interface between the macroscopic and the mesoscopic subdomains represents the major difficulty to handle [19]. The boundary conditions at this interface for the DE must be consistent with the boundary conditions for the RTE, which is not an easy task and can lead to strong error [15]. In our previous work, a dynamic multi-scale model based on the domain-decomposition strategy has been proposed [20] which allows to overcome the interface treatment difficulties. In this model, the DE and the RTE are coupled through the equations instead of being coupled through a geometric interface. A buffer zone is introduced between the kinetic and the diffusive subdomain, and the coupling is handled inside this buffer zone. Note that no boundary conditions are needed for the DE with this model.

Another possible approach for dealing with multi-scale problems is based on the so-called micro–macro formulation [21–23]. In this formulation, the mesoscopic unknown is split into a mesoscopic and a macroscopic components, and a two-way coupled system of equations is obtained for these two components. The micro–macro model is equivalent to the initial kinetic equation, unlike domain decomposition methods. During the derivation of the macroscopic equation from the kinetic equation, the exact boundary conditions are generally lost, and artificial boundary conditions may be needed. However, it is possible to find an alternative decomposition which matches the exact boundary conditions as proposed in [24]. The principal problem of the micro–macro formulation is to deal with the coupling between the macroscopic and the kinetic equations. Indeed, the mesoscopic equation cannot be solved easily and efficiently with a Monte Carlo method because of the coupling with the macroscopic equation.

In this work, a new multi-scale hybrid transport-diffusion (HTD) model is proposed for radiative transfer. In this model, similarly to the micro–macro model, the radiative intensity is decomposed into a macroscopic and a mesoscopic components, leading to a macroscopic and a mesoscopic equations. The major difference with the micro–macro model is that the two equations are one-way coupled instead of being two-way coupled. The macroscopic equation does not depend on the resolution of the mesoscopic equation and can be solved independently. Therefore, the macroscopic diffusion equation is first solved on the whole time and space intervals, and then the mesoscopic equation is solved with a source term depending on the unknown of the macroscopic equation previously computed. The model obtained enables to reconstruct the solution of the RTE by adding the solution of the diffusion equation and the solution of the mesoscopic equation. In particular, the exact boundary conditions are conserved.

In the next section, the models for radiative transfer are presented and illustrated in a multi-dimensional case (Section 2.3). In Section 3, the HTD model is tested with the MCM in a stationary test case related to concentrated solar