Journal of Computational Physics 275 (2014) 667–695

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

A finite volume method for a two-phase multicomponent polymer flooding

Sudarshan Kumar K, Praveen C, G. D Veerappa Gowda

TIFR Centre for Applicable Mathematics, Bangalore 560065, India

ARTICLE INFO

Article history:
Received 19 February 2013
Received in revised form 9 July 2014
Accepted 11 July 2014
Available online 17 July 2014

Keywords:
Conservation laws
Discontinuous flux
Finite volume
Polymer flooding
Multicomponent
Riemann problems

ABSTRACT

Multicomponent polymer flooding used in enhanced oil recovery is governed by a system of coupled non-strictly hyperbolic conservation laws. In the presence of gravity, the flux functions need not be monotone and hence designing Godunov type upwind schemes is difficult and computationally expensive. To overcome this difficulty, we use the basic idea of discontinuous flux to reduce the coupled system into an uncoupled system of scalar conservation laws with discontinuous coefficients. For these scalar equations we use the DFLU flux developed in [5] to construct a second order scheme. The scheme is shown to satisfy a maximum principle and the performance of the scheme is shown on both one and two dimensional test problems.

1. Introduction

The numerical simulation of two phase flow in porous media plays a key role in many engineering applications such as oil-recovery [6,7,33], environmental remediation [21] and water management in polymer electrolyte fuels cells [31]. Enhanced oil recovery involves the pumping of water into an oil reservoir to displace the residual oil towards a recovery well. In this paper we consider the multi-dimensional simulation of two phase (water and oil) flow in a heterogeneous porous medium arising in enhanced oil-recovery process. We have assumed that m chemical components are dissolved in the aqueous phase. These components could be different polymers that have different influence on the fluid properties like viscosity. The increased viscosity of the aqueous phase reduces fingering instabilities and hence improves the efficiency of oil recovery.

For simplicity we take the spatial domain $\Omega = [0,1] \times [0,1]$ as the two dimensional reservoir. Let $s \in [0,1]$ denote the saturation of aqueous phase and $c = (c_1, c_2, \dots, c_m) \in [0,1]^m$ denote the concentration of the polymers dissolved in the aqueous phase. Then in the absence of capillary pressure the governing equations form a system of $(m+1)$ hyperbolic conservation laws [24,25] given by

$$\begin{aligned} & s_t + \nabla \cdot F(s, c_1, c_2, \dots, c_m, x) = 0 \\ & \big(sc_l + a_l(c_l)\big)_t + \nabla \cdot \big(c_l F(s, c_1, c_2, \dots, c_m, x)\big) = 0, \quad l = 1, 2, \dots, m \end{aligned} \tag{1.1}$$

where $(x,t) \in \Omega \times (0,\infty)$, $a_l : [0,1] \to \mathbb{R}$ are given smooth functions modeling the adsorption process on the porous medium, and the flux $F : [0,1] \times [0,1]^m \times \Omega \to \mathbb{R}^2$ is given by $F = (F_1, F_2)$ where

E-mail addresses: sudarshan@math.tifrbng.res.in (Sudarshan Kumar K), praveen@math.tifrbng.res.in (Praveen C), gowda@math.tifrbng.res.in (G. D Veerappa Gowda).

http://dx.doi.org/10.1016/j.jcp.2014.07.014
0021-9991/

$$F_1(s,c,x) = v_1(x) f(s,c), \quad f(s,c) = \frac{\lambda_w(s,c)}{\lambda_w(s,c)+\lambda_o(s)}, \tag{1.2}$$

$$F_2(s,c,x) = \left[v_2(x) - (\rho_w - \rho_o) g \lambda_o(s,c) K(x)\right] f(s,c). \tag{1.3}$$

Here ρ_w, ρ_o are the densities of water and oil respectively and g is the acceleration due to gravity. The quantities λ_w and λ_o are the mobilities of the water and oil phase respectively and $v = (v_1, v_2) \in \mathbb{R}^2$ is the total velocity given by the Darcy law [16]

$$v = -\left((\lambda_w + \lambda_o) K(x) \frac{\partial p}{\partial x_1}, (\lambda_w + \lambda_o) K(x) \frac{\partial p}{\partial x_2} + (\lambda_w \rho_w + \lambda_o \rho_o) g K(x)\right) \tag{1.4}$$

where $p : \Omega \to \mathbb{R}$ is the pressure and $K : \Omega \to [0, \infty)$ is the permeability of the rock which can be discontinuous in x. A commonly used model for the mobilities is given by

$$\lambda_w(s,c) = \frac{s^2}{\mu_w(c)}, \qquad \lambda_o(s) = \frac{(1-s)^2}{\mu_o} \tag{1.5}$$

where μ_w, μ_o are the viscosities of water and oil respectively and $\mu_w = \mu_w(c)$ which is increasing in each of its variable c_i.

If we assume incompressibility of the flow and if there are no sources, then the velocity is governed by

$$\nabla \cdot v = 0 \quad \text{in } \Omega \tag{1.6}$$

with some suitable boundary conditions for pressure on $\partial\Omega$. For instance in the inlet part of the boundary, water is pumped in at high pressure $p = p_I$ while a lower pressure $p = p_O$ is maintained on outlet, see Fig. 13. On the remaining part of the boundary, the normal velocity is set to zero, which gives a Neumann boundary condition on pressure. Eqs. (1.1)–(1.6) form a system of coupled algebraic–differential equations and no time derivative is involved in Eq. (1.6).

In the absence of polymer flooding or equivalently if the flux function is independent of c, Eq. (1.1) reduces to a scalar equation. This situation has been studied in [28] by using a fast marching method and in [27] by using semi-Godunov scheme. In [17] two-phase flow problems are studied by using gradient schemes. It is well known that fingering instabilities develop when less viscous aqueous phase displaces the more viscous oil phase [13]. Due to the fingering process, water can reach the recovery well before all the oil has been displaced which leads to inefficient recovery process. With the addition of some polymer, viscosity of water increases and the fingering effects reduces which leads to an efficient oil-recovery. In the presence of the polymer concentration c, the system of Eqs. (1.1) becomes coupled and non-strictly hyperbolic. When the concentration c is smooth function of space, existence and uniqueness theory is established in [34] but we deal here with the case when c need not be smooth. For this system, developing a Godunov type upwind scheme is difficult as it needs the exact solution of Riemann problems. The presence of gravity leads to non-monotone fluxes; this combined with heterogeneity of the porous medium makes the exact Riemann solution to be computationally expensive.

By using the idea of discontinuous flux we reduce the system to a set of uncoupled scalar equations. These scalar equations have fluxes which are discontinuous in the space variable, for which we develop a Godunov scheme following the approach in [3,5]. The resulting scheme is referred to as DFLU scheme. This approach does not require detailed information about the eigenstructure of the full system. Also in [27], the idea of discontinuous flux is used to study a coupled system arising in three-phase flows in porous media and shown its successfulness. Scalar conservation laws with discontinuous flux have been studied by many authors [2,4,10,9,11,14,15,18,22,26,30]. In particular, a Godunov type finite volume scheme is proposed in [3] and convergence to a proper entropy solution is proved, provided the flux functions satisfies certain conditions like in Section 2. The DFLU scheme for system of equations in the one-dimensional case is introduced in [5]. In the present work we are extending DFLU to a multi dimensional case including the effect of gravity, spatial heterogeneity and arbitrary number of polymer components and develop a second order scheme. We compare numerical results with Godunov scheme which is obtained by solving exact Riemann problem and Upstream mobility (UM) scheme which is an ad-hoc scheme used by petroleum engineers. We point out that, in general UM flux may lead to a wrong solution, see [32].

The paper is organized as follows. In Section 2 we discuss solution to the Riemann problem and explain the construction of first and second order DFLU scheme. The second order scheme is constructed by using a reconstruction process in space variable together with slope limiter and a strong stability preserving Runge–Kutta scheme in the time variable [20]. In Section 3 numerical results for one dimensional case are presented. In Section 4 two dimensional problem is introduced and the one dimensional scheme is extended to two dimensions. The resulting schemes are shown to respect a maximum principle. In Section 5 two dimensional numerical results are shown for various test cases.

2. One dimensional problem

The system of $(m+1)$ equations in one-dimension in the presence of gravity is given by

$$\begin{aligned} & s_t + \frac{\partial}{\partial x} F(s, c_1, c_2, \ldots, c_m, x) = 0 \\ & \left(s c_l + a_l(c_l)\right)_t + \frac{\partial}{\partial x} c_l F(s, c_1, c_2, \ldots, c_m, x) = 0, \quad l = 1, 2, \ldots, m \end{aligned} \tag{2.1}$$