Journal of Computational Physics 271 (2014) 281–305

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Numerical treatment of wave breaking on unstructured finite volume approximations for extended Boussinesq-type equations

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ARTICLE INFO

Article history:
Received 21 May 2013
Received in revised form 12 November 2013
Accepted 18 January 2014
Available online 23 January 2014

Keywords:
Boussinesq-type equations
Wave breaking
Finite volumes
Unstructured meshes

ABSTRACT

A new methodology is presented to handle wave breaking over complex bathymetries in extended two-dimensional Boussinesq-type (BT) models which are solved by an unstructured well-balanced finite volume (FV) scheme. The numerical model solves the 2D extended BT equations proposed by Nwogu (1993), recast in conservation law form with a hyperbolic flux identical to that of the Non-linear Shallow Water (NSW) equations. Certain criteria, along with their proper implementation, are established to characterize breaking waves. Once breaking waves are recognized, we switch locally in the computational domain from the BT to NSW equations by suppressing the dispersive terms in the vicinity of the wave fronts. Thus, the shock-capturing features of the FV scheme enable an intrinsic representation of the breaking waves, which are handled as shocks by the NSW equations. An additional methodology is presented on how to perform a stable switching between the BT and NSW equations within the unstructured FV framework. Extensive validations are presented, demonstrating the performance of the proposed wave breaking treatment, along with some comparisons with other well-established wave breaking mechanisms that have been proposed for BT models.

1. Introduction

Mathematical and numerical modeling of non-linear wave transformations and their corresponding process has received much attention in the past few decades and has been one of the most interesting and active research fields in coastal engineering for simulating near-shore dynamics. Important issues one has to consider include, the validity of a mathematical model in near-shore zones as well as in deeper waters, representation of wave breaking, frequency dispersion and accurate numerical treatment of natural topographies and wetting/drying processes. Among these key issues, the natural phenomenon of wave breaking is of fundamental significance, playing a key role in the near-shore dynamics. In terms of the mathematical modeling, important physical effects associated with non-linear transformations of sea waves in near-shore regions can be described by Boussinesq-type (BT) equations. BT equations are more appropriate for describing flows in deeper waters where frequency dispersion effects may become more important than non-linearity. BT equations introduce dispersion terms in the modeling thus being more suitable in waters where dispersion begins to have an effect on the free surface.

Following from the seminal work of Boussinesq [12], the first set of extended BT equations was derived by Peregrine [51], under the assumption that non-linearity and frequency dispersion are weak and they are limited to relatively shallow

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0021-9991/$ – see front matter
http://dx.doi.org/10.1016/j.jcp.2014.01.030

water due to the weak dispersion. Subsequent attempts to extent the validity and applicability of these so-called standard Boussinesq equations have been successful. Madsen and Sørensen [46] and Nwogu [48] have extended their validity by giving a more accurate representation of the phase and group velocities in intermediate waters, closely relating to linear wave theory. Furthermore, significant effort has been made in recent years into advancing the non-linear and dispersive properties of BT models by including high order non-linear and dispersion terms, we refer for example to [8,28,40,9,10], among others, which in turn are more difficult to integrate and thus require substantially more computational effort in their numerical integration. For a very recent comprehensive review on the theory, numerics and applications of BT models we refer to the review work in [13].

Until recently, the predominant method for the numerical solution of BT equations was the finite difference (FD) method, we refer for example to [48,78,26,45,59] among many others. Finite volume (FV) methods have gained a lot of popularity the last few years. The application of the FV methodology to the BT equations is not straightforward due to the presence of the dispersive terms, therefore hybrid solutions, coupling the FV and the FD methods have been proposed for 1D BT equations, please refer, for example, in [25,11,17,35,56,42,59,60,63,10,69]. Further, this numerically hybrid approach has been extended to two space dimensions, for example in [38,72,71], but for uniform structured grids, restricting the modeling when dealing with 2D irregular geometries, similarly to the FD method. Asmar and Nwogu [24] proposed a FV scheme for Nwogu's BT equations [48] on unstructured meshes with a staggered placement of the variables. Recently Kazolea et al. [36] introduced, for the first time, the development and application of a conservative higher-order FV scheme for unstructured meshes, that exploits the advantages of the FV approach and incorporates state of the art discretizations for the topography and wet/dry front treatment.

Wave breaking has been incorporated into BT models by means of different artificial techniques. *The surface roller model* [58,43,44,64], *the vorticity model* and *the eddy viscosity model* [37,83,56,19,34] are three types of additional momentum dissipation methods. In *eddy viscosity* models dissipation due to turbulence generated by wave breaking and bore propagation is treated by a diffusion term in the momentum conservation equation in order to prevent numerical instabilities resulting from frequency and amplitude dispersion. The amount of dissipation is governed by the value of the eddy viscosity coefficient, which is expressed in terms of a mixing length parameter and is calibrated with experimental data, while a breaking criterion is used to decide exactly where and when the dissipation becomes active [20]. Heitner and Housner [31] proposed an eddy viscosity model to dissipate energy for breaking waves. Energy loss is limited to the front face of waves where the change of wave properties exceeds a certain criteria. Zelt [83] treated wave breaking similarly in a Lagrangian Boussinesq model to simulate solitary wave breaking and run-up. The same model used from Wei and Kirby [79]. Karambas and Koutitas [34] used also an eddy viscosity mechanism with the limitation that the formulation was not momentum preserving and the setup prediction in the inner surf zone (in the investigation of the performance for periodic waves) was very poor. Kennedy et al. [37] followed [31] and [83] but with extensions to provide a more realistic description of the initiation and cessation of wave breaking and were able to adequately reproduce wave height and setup for regular waves breaking on planar beaches. The largest disadvantage in that formulation is that, in some special cases, such as stationary hydraulic jumps, wave breaking initiation is not recognized. Additionally, Cienfuegos et al. [19] showed that Kennedy et al.'s eddy viscosity breaking model could hardly predict simultaneously accurate wave height and asymmetry along the surf zone. Lynett [41] used the eddy viscosity model of Kennedy et al. [37] with some modifications which regards the manner in which a breaking event is initiated and reformulations of the model's thresholds in terms of the total water depth H. Recently, Roeber et al. [56] adapt the approach of [31,37] and presented an eddy viscosity approach consistent with the conservative formulation of the BT equations of Nwogu [48] in 1D, to account for breaking waves in the surf zone.

Other methods are based on the *surface roller* concept introduced by Svendsen [66]. These methods, like the eddy viscosity ones, add a flux gradient to the BT momentum equation [58,64] but this approach stems from different hypothesis and ideas than those on the eddy viscosity ones. The added term depends on the dynamically determined roller thickness, the mean front slope of the breaker and other parameters which must be tuned during the numerical implementation. The roller approach has been improved by [43,44] and by [77,33]. Recently Cienfuegos et al. [17–19] considered the wave breaking energy dissipation through extra terms added both in mass and momentum equations.

The last few years, a new approach has emerged in which one simply (under certain conditions) turns off the dispersive terms in the region where breaking occurs, please refer to [71,72,55,35,11,9,69] among others. This formulation is based on the assumption that BT equations automatically degenerate into non-linear shallow water (NSW) equations as dispersive terms become negligible compared to non-linear terms. The idea introduced in [71–73], for the equations of Madsen and Sørensen [46], is to exploit the shock-capturing advantages of the FV scheme within the framework of BT modeling in order to simulate discontinuous phenomena such as wave breaking and run-up. More recently, the same rational was applied for a non-hydrostatic, near-shore wave model in [61]. These models take advantage of the fact that in shallow water, the NSW equations (under the FV framework) have the ability to naturally embody bore propagation and the related energy dissipation. This feature is interesting because of the similarity between spilling breakers and bores [52,5]. Borthwick et al. [11] introduced the above idea using as an indicator criterion one similar to the criterion used by Kennedy et al. [37]. In [71,72] a simple criterion (developed on a physical basis) introduced in the numerical scheme to establish which set of equations should be solved in each computational cell and recently they applied an extended version of their hybrid model, including an additional criterion for the switch back to the BT equations, to describe the transformation of irregular waves [74]. Roeber et al. [55] utilize the local momentum gradients as an indicator for deactivation of the dispersive terms. Among the works that have followed similar reasoning are also [69,9,59,11,50,10]. However, the treatment proposed in all