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A three-dimensional parametric mesher with surface boundary-layer capability

R. Aubry [a,*], B.K. Karamete [a], E.L. Mestreau [a], S. Dey [b]

[a] Sotera Defense Solutions, 1501 Farm Credit Drive, Suite 2300, McLean, VA 22102-5011, USA
[b] US Naval Research Laboratory, Code 7131, 4555 Overlook Ave SW, Washington, DC 20375, USA

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ABSTRACT

A novel parametric surface meshing technique is presented. Its distinctive feature relies on successive approximations of the CAD geometry through a hierarchical process where geometric information is gathered incrementally. A detailed review of zero- and first-order surface approximations and their impact on parametric surface meshing algorithms is performed. The proposed approach emphasizes the use of three-dimensional information in order to be as independent as possible of the parametrization to overcome limitations of meshing purely in the parametric plane. The presented technique includes semi-structured boundary-layer surface mesh generation which is a critical capability for accurate solutions to flows around geometries that have leading edge features. Numerous examples illustrate the method's robustness and ability to high-quality meshes for complex CAD geometries.

1. Introduction

Physics-based computational techniques are used increasingly in all phases of the design of complex engineering systems. In the civilian realm this includes the design of civilian airplanes and automobiles, and in the defense realm it is used for the design of military aircrafts, ships, submarines, radars and other defense systems. For complex three-dimensional structures and domains, computer-aided-design (CAD) systems are used to model the geometry in which three-dimensional surfaces are routinely represented mathematically as non-uniform-rational-B-splines or NURBS [1,2]. Computational methods based on the finite element or the finite volume approach solve the modeled governing differential equations. Both require generation of a discrete representation of the geometry of the problem domain often called a grid or mesh. Surface mesh generation is a critical step in the whole meshing process as it is needed for surface-discretization-based formulations and forms the starting point for the volume meshing process.

For problems that reveal natural anisotropy, such as those involving viscous flows around bodies, it is important to produce meshes that are stretched in the boundary-layer region. Boundary-layer meshes are required in the volumetric region in the vicinity of the boundary of the structure; they are also needed on the surface mesh near geometric features such as the leading edge of an airfoil or hydrofoil.

This work is part of a broader geometry and meshing software platform where different CAD kernels and meshers communicate through application programming interfaces (API) as plugins. Since geometry representation for different CAD systems may be very different and in fact not known explicitly, the mesher is agnostic of the particulars of a kernel as long as the kernel is able to provide needed geometric information through the application programming interfaces (APIs) as implemented for a given kernel. In such a setup, it is very important that the mesh generation approach be very robust.

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[*] Corresponding author.

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Our interest in this paper is a surface meshing algorithm that is:

- robust in its handling of both complex CAD-based and discrete geometry representation,
- produces high-quality meshes for complex geometric shapes with smooth sizing gradation control, and
- is able to generate surface boundary layer meshes around selected geometry features.

In the literature two broad categories of surface mesh generation methods may be identified. Some approaches are purely *parametric* in that the mesh is generated entirely in the two-dimensional parametric space and is then projected to its three-dimensional shape in an a-posteriori manner based on the mapping defined between the parametric domain and the three-dimensional range. Other approaches work directly in the three-dimensional space. For example, Peraire et al. [3] propose a method using an anisotropic advancing front, where stretching is taken into account through coordinate rotations aligned with the stretching interpolated from a background grid. A regular element is then generated in that space and mapped back to the parametric plane thereafter. Tristano et al. [4] advocate an anisotropic advancing front method in the parametric plane. However, the information is not purely planar as a three dimensional size is stored and used for local queries, complementing the two-dimensional metric. Angles are evaluated in the three-dimensional space. The computation of the optimal point in the Riemannian space is also provided without relying on the spectral decomposition of the metric. Their front strategy consists of sorting the front edges with respect to their three-dimensional size. In Rypl et al. [5], the geometry is represented through bicubic Bezier patches. Singularities in the parametrization are tackled through evaluation in the vicinity of purely singular points, where the tangent plane is not well defined. An advancing front technique is performed only in the parametric space. However, some parts of the optimization process take place in the three dimensional space. In Lee [6], a pure two-dimensional anisotropic advancing front is used without any reference to the three-dimensional space. Guan et al. [7] extend the advancing front technique to take into account parametric surfaces through a point and edge shift operator to locally simulate the three-dimensional proximity in the two-dimensional space. Cuillière [8] also mentions briefly the difficulty associated with closed surfaces. An advancing front technique is also proposed that takes into account the metric of the first fundamental form. Reliance on three-dimensional information is not reported. The INRIA gamma project has proposed an original approach for parametric surfaces that relies on an anisotropic Delaunay insertion in Borouchaki et al. [9]. In Borouchaki et al. [10], the anisotropic Delaunay kernel is coupled with an advancing front point placement. The notion of geometric mesh is emphasized in Laug [11]. Finally, the work of Borouchaki et al. [12] tries to remove the strong constraints introduced by the geometry in case of small geometry entities.

Parametric surface meshers are often considered to be more robust and faster than their three-dimensional counterparts. However, this is not always the case. Parametric meshers rely heavily on differential geometry [13–15] and as noticed by Jiao et al. [16], the discretization of the continuous differential geometry formulae can lead to difficulties, or even inconsistencies depending on the discretization process. This leads to a lack of robustness. From the standpoint of computational efficiency, instead of evaluating the true three-dimensional length (area), parametric meshers evaluate a metric associated with the mapping. The cost of evaluating the mapping functions and their needed derivatives (up to first order without curvature and up to second order for curvature evaluation) for the metric can be non-trivial for complex mapping functions used with trimmed CAD surface representations. Additionally, there are several subtle, but practical issues associated with a purely parametric approach which are rarely reported in the literature. These include:

- When a specific mesh sizing information is prescribed, it typically represents the length of the straight edges in the final three-dimensional mesh. For mesh generators based on local operation, such as splits, swaps and collapses, a crucial task consists of computing the mesh edge length. A purely parametric approach, assuming an infinitely precise metric computation, evaluates the length of the curved edge on the surface when the relevant quantity is the length of the straight edge segment as depicted in Fig. 1.1. In the limit of infinite refinement, the curve length and the straight mesh edge length will converge to the same value; however, for practical mesh sizes on surfaces and edges with large curvature variations a purely parametric approach may produce meshes that will not satisfy the prescribed three dimensional sizing. The same phenomenon may happen for a highly distorted parametrization.
- As noted in Tristano et al. [4], the parametrization of CAD surfaces in general, and NURBS surfaces in particular, is most of the time not uniform. Advancing front based methods are greedy by nature, trying to generate the best element locally to achieve the best mesh quality globally. As a matter of fact, the creation of a bad element will have consequences in the generation of the future elements. The construction of an optimal triangle is therefore the crucial point of this method compared to local operations. Therefore, as illustrated in Fig. 1.2, an inaccuracy in the metric evaluation gives rise to an inaccurate three-dimensional size, causing irregularities in the final three dimensional mesh. Three-dimensional informations may require a couple of iterations for a point to converge to its optimal placement. However, the final mesh is much less dependent on the parametrization, while reducing a potentially intense final optimization stage.
- The choice of a metric implicitly embeds one or more properties of the geometric approximation. As seen thereafter, the first fundamental form allows to measures three dimensional lengths in the parametric plane and the second fundamental form measures the curvature of the surface. Both may be taken into account into a single metric. However, these properties may not be relevant from the standpoint of the validity or accuracy needs of a three-dimensional mesh. There is therefore no guarantee that a valid two-dimensional mesh based on some appropriate metric will produce a valid three-dimensional surface mesh. As noted in Laug [11], even the metric based on the second fundamental form