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A hybrid transport-diffusion method for 2D transport problems with diffusive subdomains

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ABSTRACT

In this paper we present a computational method based on the Simple Corner Balance (SCB) scheme for solving 2D transport problems in diffusive media. It utilizes decomposition of spatial domain into transport and diffusive subregions. This methodology uses the low-order equations of the Second-Moment (SM) method for the first two angular moments of the transport solution. These low-order SM equations are solved globally. The high-order transport solution is computed only in transport subregions. The transport boundary conditions at interfaces with neighbouring diffusion subregions are formulated using asymptotic analysis of SCB. We apply the quasidiffusion (Eddington) tensor to evaluate transport effects in the problem domain and determine spatial ranges of diffusive subregions. Numerical results are presented. They demonstrate the accuracy of the developed methodology for the SCB scheme.

1. Introduction

The interaction of particles with matter is described by the linear Boltzmann equation. It is a detailed conservation relation in the phase space defined by the spatial position of a particle, its direction of flight, and energy [1]. There exists an important class of particle transport and radiative transfer problems in which spatial domains contain diffusive subregions [1,2]. Such domains are characterized by small absorption and large optical thickness. This means that a particle will undergo a large number of collisions during its lifetime because the probability of scattering is far greater than the probability of absorption or leakage. In highly diffusive domains, the diffusion approximation is valid. As a result, the transport of particles can be accurately described by an approximate mathematical model based on the P_1 equations [1]. These equations are formulated for the first two angular moments of the transport solution and hence have lower dimensionality compared to the transport equation. An asymptotic analysis of the transport equation in a diffusive medium also shows that its solution can be well approximated by the solution of the diffusion equation [3–5]. The solution of the transport equation is a function of a large number of independent variables. It is feasible to decrease dimensionality of the original problem by solving a diffusion problem in diffusive subdomains instead of the transport equation. In order to take advantage of this option, it is necessary to decompose the spatial domain of the problem into transport and diffusive subregions and develop a methodology for solving the global problem with domain decomposition.

There is a group of numerical methods for solving transport problems that applies this kind of domain decomposition. The general approach is (i) to formulate transport and diffusion problems locally in each subregion of corresponding kind and (ii) to define coupling conditions for solutions at interfaces of the subregions [6–15]. The resulting approximate interface

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relations include boundary conditions for the diffusion equation and boundary conditions for the transport equation. They are derived by means of an asymptotic diffusion analysis of the transport equation.

Another group of computational methods is based on low-order equations for the moments of the transport solution that exactly account for transport effects and give rise to the diffusion equation in diffusive regions [16,17]. This low-order problem is solved globally in the entire domain. The solution of the transport equation is computed in transport subdomains. The interface conditions are required only for the transport equation at boundaries of transport subdomains. This methodology was developed and applied for 1D transport problems [17]. Its purpose is to improve computational efficiency by calculating approximately a discrete solution of a given transport discretization scheme and to minimize error caused by approximation and decomposition of spatial domain into transport and diffusion subregions.

To describe the basic idea behind this methodology, let us consider a transport discretization scheme

$$\mathrm{L}_h \psi_h = \mathrm{S}_h \psi_h + \mathrm{Q}_h \quad \text{in } G_h \times \omega_h, \tag{1}$$

$$\varphi_h = \mathrm{M}_h \psi_h, \tag{2}$$

where G_h is the spatial grid over the entire domain of the problem, ω_h is the angular grid, L_h and S_h are discrete loss and scattering operators, respectively, that define the transport scheme on the given grid in space (G_h) and angle (ω_h), Q_h is the vector of an external source, ψ_h is the vector of the discrete transport solution, φ_h is the vector of discrete angular moments of ψ_h, for instance, the scalar flux and current, M_h is the discrete angular moment operator. A wide class of computational transport methods for solving Eq. (1) is based on linear acceleration schemes [18]. These methods can be presented by the following system of equations:

$$\psi_h = \mathrm{L}_h^{-1} \mathrm{P}_h \varphi_h + \mathrm{L}_h^{-1} \mathrm{Q}_h \quad \text{in } G_h \times \omega_h, \tag{3}$$

$$\mathrm{D}_h \varphi_h = \mathrm{Q}_h + \mathrm{C}_h[\psi_h] \quad \text{in } G_h. \tag{4}$$

Eq. (4) is a low-order problem for the moments of the angular flux. D_h is the discrete operator of the diffusion equation algebraically consistent with the transport discretization scheme (1). C_h is the closure term. Note that if the closure term C_h is set to zero then Eq. (4) is reduced to the discretized diffusion equation. The discrete transport equation (3) is coupled to Eq. (4) by means of corresponding scattering operator P_h. The system of Eqs. (3) and (4) is equivalent to Eq. (1).

In the proposed method the transport problem is solved by means of the same transport discretization scheme on a subset of spatial subdomains $\tilde{G}_h$

$$\tilde{\psi}_h = \mathrm{L}_h^{-1} \mathrm{P}_h \tilde{\varphi}_h + \mathrm{L}_h^{-1} \mathrm{Q}_h \quad \text{in } \tilde{G}_h \times \omega_h \ (\tilde{G}_h \subset G_h) \tag{5}$$

with approximate boundary conditions at interface $\partial \tilde{G}_h$ with neighbouring diffusion subdomains. The spatial grid in transport subdomains is identical to the grid for the original transport problem (1). The moments of the angular flux are computed from the low-order problem defined in the entire spatial domain

$$\mathrm{D}_h \tilde{\varphi}_h = \mathrm{Q}_h + \tilde{\mathrm{C}}_h[\tilde{\psi}_h] \quad \text{in } G_h \tag{6}$$

with a modified closure term $\tilde{\mathrm{C}}_h$. The problem (5) and (6) is an approximation to problem (3) and (4) and hence to Eq. (1) on the given phase-space grid. Its solution ($\tilde{\psi}_h$ and $\tilde{\varphi}_h$) is an approximation of the discrete transport solution (ψ_h and φ_h) computed in the entire spatial domain. We note that $\tilde{\psi}_h$ is calculated from Eq. (5) and hence defined only on a set of transport subdomains. In diffusion subregions it can be approximated by P_1 expansion using the angular moments $\tilde{\varphi}_h$ from Eq. (6). Thus, computational costs are reduced because the high-order transport problem is not solved in diffusion subdomains. The effect on computations is proportional to the number of spatial cells where the transport equation is not solved. The error of the approximate solution is determined, for instance, by

$$\|\tilde{\varphi}_h - \varphi_h\|. \tag{7}$$

It can be decreased by improving interface transport boundary conditions and optimizing spatial ranges of transport subdomains. We note that there is conflict between accuracy and efficiency. The deeper subdomain interfaces in diffusion regions the smaller the error. However, this increases a domain where the transport equation is solved and hence reduces efficiency of computations. On the other hand, moving interfaces into transport regions diminishes amount of calculations but it increases the error in $\tilde{\varphi}_h$. The reason is that boundary conditions at interfaces become less accurate.

In this paper we extend this approach to multidimensional transport problems and use it to develop a computational method with decomposition of the problem domain into transport and diffusive subdomains for the Simple Corner Balance (SCB) scheme [19] in 2D Cartesian geometry. This method uses low-order equations of the Second-Moment (SM) method [20,18] that are discretized consistently with the SCB scheme [21]. The low-order SM (LOSM) problem couples naturally transport and diffusion problems in corresponding subdomains in a low-order space. It also provides necessary acceleration of transport iterations. In highly diffusive spatial regions, the consistently discretized LOSM equations limit to the intrinsic diffusion discretization of the SCB scheme. We note that the SM method is similar to the Diffusion Synthetic Acceleration (DSA) method [18], except that the LOSM problem is formulated for the solution itself while the low-order DSA equations are for iterative corrections (errors).