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Multiscale/fractional step schemes for the numerical simulation of the rotating shallow water flows with complex periodic topography

F. Jauberteau [a,*], R.M. Temam [b], J. Tribbia [c]

[a] Université de Nantes, Laboratoire de Mathématiques, CNRS UMR 6629, BP 92208, 44322 Nantes, France
[b] The Institute for Applied Mathematics and Scientific Computing, Indiana University, Rawles Hall, Bloomington, IN 47405, USA
[c] National Center for Atmospheric Research (NCAR), Climate and Global Dynamics Division, Boulder, CO 80305, USA

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ABSTRACT

In this paper, we study several multiscale/fractional step schemes for the numerical solution of the rotating shallow water equations with complex topography. We consider the case of periodic boundary conditions (f-plane model). Spatial discretization is obtained using a Fourier spectral Galerkin method. For the schemes presented in this paper we consider two approaches. The first approach (multiscale schemes) is based on topography scale separation and the numerical time integration is function of the scales. The second approach is based on a splitting of the operators, and the time integration method is function of the operator considered (fractional step schemes). The numerical results obtained are compared with the explicit reference scheme (Leap-Frog scheme). With these multiscale/fractional step schemes the objective is to propose new schemes giving numerical results similar to those obtained using only one uniform fine grid $N \times N$ and a time step Δt, but with a CPU time near the CPU time needed when using only one coarse grid $N_1 \times N_1$, $N_1 < N$ and/or a time step $\Delta t' > \Delta t$.

1. Introduction: motivation of the problem

In full atmospheric models the influence of topography on temperature and precipitation is important. For example it is crucial to take correctly into account small scales of complex topography in order to obtain a good estimation of the total precipitation (see Smith [32], Smith and Barstad [33]) and the proper specification of temperature at the height of the topography is critical for the determination of snowpack and water resources. Primarily for these two reasons regional models are of use in climate simulations, so that the local impact of high resolution topographic forcing can be incorporated in simulations of seasonal climate and climate change predictions.

Despite this need, few studies have examined the efficacy of alternative representations of the dynamic effects of small scale topography on precipitation, the most notable exceptions being (Smith and Barstad [33], Leung and Ghan [34]). This is in spite of the fact that at least one component of the topographic forcing is known to high accuracy, the topography itself.

In recent years several new techniques have arisen within the applied mathematics community, that are able to utilize small scale information in a self-consistent fashion to approximate the effects of small scales on the temporal tendencies of

* Corresponding author.
E-mail addresses: francois.jauberteau@univ-nantes.fr (F. Jauberteau), temam@indiana.edu (R.M. Temam), tribbia@ucar.edu (J. Tribbia).

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the large scale. We report here on nascent attempts to apply these new methods to the problem of topographically forced flow and its dynamical and thermodynamical effects on the large scale. To test these methods we first use the simplest geophysical model that can adequately expose the dynamical effects of topography, a rotating shallow water system.

For the application of multiscale methods in meteorological modeling we can also refer to Vater et al. [37].

The shallow water equations with Coriolis forces are used in atmospheric modeling as a simplification of the primitive equations of atmospheric flow. In realistic applications there is variable bottom topography that adds a source term to the shallow water equations, similar to the Coriolis force. To take into account the small scales in the numerical computation of the solution of the shallow water problem with complex topography, it is necessary to retain sufficient modes (points) in the two spatial directions, thus increasing the computational time.

This article can be considered as a continuation of Dubois et al. [18] in which we proposed new numerical schemes to compute the rotating shallow water equations on a flat bottom. The topography Z_S is introduced in the present study, which produces a perturbation term in the height equation (see the second equation (1.1)), with a separation of scales in the topography to distinguish between the large and small scales of the terrain (see (1.2) below). From the algorithmic point of view the reference scheme is the same, namely a spectral Galerkin method with an explicit time scheme. A more detailed comparison of the multilevel algorithms considered here and in Dubois et al. [18] appears below. Besides, as explained above (and below), we want here to produce decomposition methods based on the scales of the topography.

These new schemes are based on a scale separation (multiscale schemes) or on a splitting of the operators (fractional step schemes). The objective of this work is to propose new schemes giving numerical results close to those obtained using only one uniform fine grid $N \times N$ and a time step Δt, but with a CPU time comparable to the CPU time needed when using only one coarse grid $N_1 \times N_1$, $N_1 < N$ and/or a time step $\Delta t' > \Delta t$.

Let us consider the rotating shallow water equations with topography. The equations are written as follows:

$$\begin{cases} \dfrac{\partial \mathbf{u}}{\partial t} + (\boldsymbol{\Omega} \times \mathbf{u}) + f\mathbf{u}^{\perp} + \nabla\left(gh + \dfrac{1}{2}|\mathbf{u}|^2\right) = \mathbf{0}, \\ \dfrac{\partial h}{\partial t} + \operatorname{div}(h\mathbf{u}) = \operatorname{div}(Z_S\mathbf{u}), \end{cases} \tag{1.1}$$

where $\mathbf{u} = (u, v)^{\top}$ is the velocity field, $\mathbf{u}^{\perp} = (-v, u)^{\top}$ the orthogonal velocity field, $\boldsymbol{\Omega} = \nabla \times \mathbf{u}$ the vorticity vector, h the height of the free surface, Z_S is the topography, f is the Coriolis force, g the gravity and $|\cdot|$ the Euclidean norm.

With a multiscale scheme, we intend to use a scale separation based on the scales of the terrain, and then to adapt the equations as function of the scales.

Let us write the scale decomposition of the topography:

$$Z_S = \overline{Z}_S + Z'_S, \tag{1.2}$$

where $\overline{Z}_S$ (resp. Z'_S) corresponds to the large (resp. small) scales of the terrain. The time evolution equation of h in (1.1) can be rewritten since it is linear in Z_S:

$$\frac{\partial h}{\partial t} + \operatorname{div}(h\mathbf{u}) = \operatorname{div}(\overline{Z}_S\mathbf{u}) + \operatorname{div}\big(Z'_S\mathbf{u}\big). \tag{1.3}$$

Here we consider that $\overline{Z}_S$ is associated with a large scale size in space $\Delta\mathbf{x}$ and Z'_S with a small scale size $\frac{\Delta\mathbf{x}}{\eta}$, with $\eta \gg 1$, in order to take into account the small scales of the terrain. Moreover we retain the hypothesis that Z'_S is small in comparison with $\overline{Z}_S$ (this is the case, for example, in the scale separation using Fourier expansion). Let us consider the following model for Z'_S:

$$Z'_S(\mathbf{x}) = \varepsilon\overline{Z}_S\big(\mathbf{x}^*\big), \tag{1.4}$$

where $\varepsilon \ll 1$, $\mathbf{x}$ is the spatial scale associated with $\Delta\mathbf{x}$, $\mathbf{x}^* = \eta\mathbf{x}$, ε and η independent.

From the scale separation $Z_S = \overline{Z}_S + Z'_S$ we can consider a decomposition on h:

$$h = \bar{h} + h', \tag{1.5}$$

where $\bar{h}$ is associated with $\overline{Z}_S$ and h' is associated with Z'_S, with the following equations for $\bar{h}$ and h', deduced from (1.3):

$$\begin{cases} \dfrac{\partial \bar{h}}{\partial t} + \bar{h}\operatorname{div}(\mathbf{u}) + \mathbf{u}\cdot\nabla\bar{h} = \overline{Z}_S\operatorname{div}(\mathbf{u}) + \mathbf{u}\cdot\nabla\overline{Z}_S, \\ \dfrac{\partial h'}{\partial t} + h'\operatorname{div}(\mathbf{u}) + \mathbf{u}\cdot\nabla h' = Z'_S\operatorname{div}(\mathbf{u}) + \mathbf{u}\cdot\nabla Z'_S. \end{cases} \tag{1.6}$$

We have that $h = \bar{h} + h'$ is solution of (1.3). Using (1.4), we can rewrite (1.6) in the following manner: