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A stable second-order scheme for fluid–structure interaction with strong added-mass effects

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ABSTRACT

In this paper, we present a stable second-order time accurate scheme for solving fluid–structure interaction problems. The scheme uses so-called Combined Field with Explicit Interface (CFEI) advancing formulation based on the Arbitrary Lagrangian–Eulerian approach with finite element procedure. Although loosely-coupled partitioned schemes are often popular choices for simulating FSI problems, these schemes may suffer from inherent instability at low structure to fluid density ratios. We show that our second-order scheme is stable for any mass density ratio and hence is able to handle strong added-mass effects. Energy-based stability proof relies heavily on the connections among extrapolation formula, trapezoidal scheme for second-order equation, and backward difference method for first-order equation.

Numerical accuracy and stability of the scheme is assessed with the aid of two-dimensional fluid–structure interaction problems of increasing complexity. We confirm second-order temporal accuracy by numerical experiments on an elastic semi-circular cylinder problem. We verify the accuracy of coupled solutions with respect to the benchmark solutions of a cylinder-elastic bar and the Navier–Stokes flow system. To study the stability of the proposed scheme for strong added-mass effects, we present new results using the combined field formulation for flexible flapping motion of a thin-membrane structure with low mass ratio and strong added-mass effects in a uniform axial flow. Using a systematic series of fluid–structure simulations, a detailed analysis of the coupled response as a function of mass ratio for the case of very low bending rigidity has been presented.

1. Introduction

A stable and efficient numerical technique is essential for the study of nonlinear fluid–structure interaction (FSI) problems in the fields of aerospace [1,2], bio-engineering [3,4], civil and offshore engineering [5,6]. Two main categories of methods are available, which differ on their treatment of fluid–structure interface. In the first category, a discrete version of the interface moves in a non-conformal manner across a fixed mesh in space. This requires an additional property to capture the interface such as a level set function [7], an immersed structure [4,8], a Lagrange multiplier, fictitious domains or ghost fluid [9,10]. In the second category, both the interface and mesh in space move together keeping conformity in a body-fitted manner. The arbitrary Lagrangian–Eulerian (ALE) approach [11,12] was introduced to handle the time derivative terms on

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the time-varying fluid domain. It prevents excessive mesh distortion comparing with the pure Lagrangian approach. The capability to design high order methods makes methods of the second category of great interest for practical problems.

The ALE based FSI simulations are generally accomplished by using either partitioned or monolithic schemes. A monolithic [13–16] approach assembles the fluid and structural equations into a single block and solves them simultaneously for each iteration. These schemes lack the advantage of flexibility and modularity of using existing stable fluid or structural solvers. However, they offer good numerical stability even for problems involving very strong added mass effects. In contrast, a partitioned approach solves the fluid and structural equations in a sequential manner, facilitating the coupling of the existing fluid and structural program with minimal changes. This trait of the partitioned approach therefore makes itself an attractive option from computational point of view.

Typically, partitioned staggered schemes [17] are classified as either strongly-coupled [18] or loosely-coupled [19,20]. Loosely-coupled schemes satisfy the interface velocity continuity and traction continuity conditions in a sequential manner. These schemes often suffer from numerical instability and temporal inaccuracy caused by spurious energy production along the interface due to the time lag [21,22], and special treatments are generally required to address these issues. A variety of force corrections and structural predictors [19,23] are used to increase the numerical stability of loosely-coupled schemes. Strongly-coupled schemes typically involve predictor–corrector sub-iterations to ensure the convergence of interface properties. However, this results in increased computational cost. In several applications such as, flow through blood vessels [24], ocean current interactions with offshore risers [6], strongly-coupled schemes suffer from convergence issues due to strongly predominant added mass effects [25].

The key objective of this paper is to present an efficient second-order scheme to solve the fully coupled FSI problems which is stable for any mass density ratio. We further present an energy based stability proof for the proposed second order scheme. The scheme is based on the combined field formulation proposed in [26] which uses the ALE description for the fluid and Lagrangian description for the solid. The combined field scheme presented in [26] is first-order accurate and is motivated by the one-fluid formulation [27] for studying multiphase problems. There is no systematic study on the effects of mass ratio in the earlier work presented in [26].

The CFEI formulation is based on weak formulation of fluid–structure problem with properly chosen function spaces for the unknown variables. Both governing equations for fluid and solid are written in terms of their velocities, respectively. Solid position plays only a role of slave variable. The continuity of velocities across the fluid–structure interface is enforced in the function spaces, while the continuity of traction across the interface is enforced in the weak formulation. The definition of the function space for the velocities requires information on the solid position. Since the solid position is also an unknown quantity, decoupling of the computations of the solid position and the remaining variables is performed in this weak formulation. In the CFEI formulation, this requirement automatically leads to an explicit interface advancing for updating the solid positions and handling mesh velocity in the ALE formulation of the Navier–Stokes equations. This explicit advancing of the mesh velocity in turn requires an explicit treatment of the convective velocity in the quadratic term of the Navier–Stokes equations to achieve a desired energy balance. The other velocity in the quadratic terms is treated implicitly and this semi-implicit treatment is crucial for the stability proof as shown in [26]. Another important consequence of the semi-implicit treatment is that the CFEI formulation only requires to solve a linear system of equations per time step.

The proposed CFEI scheme is unconditionally stable with respect to mass ratios with relatively lesser number of unknowns compared to the traditional monolithic schemes. The energy-based stability proof for the second-order fully discrete scheme proposed in this paper relies heavily on the CFEI formulation and on the connections among the second-order extrapolation formula, the backward differentiation, and the trapezoidal rule. The numerical scheme preserves the energy decaying property for a fluid–structure system. The temporal accuracy of the CFEI scheme has been assessed with the aid of 2D incompressible flow interacting with structure. We show the second-order temporal accuracy for a test problem on elastic semi-circular cylinder and verify the precision with the benchmark results for the cylinder-elastic bar problem. Furthermore, we use this CFEI scheme to study the flapping dynamics involving low mass density ratio and bending rigidity modulus.

The content of this paper is organized as follows. In Section 2, we start with the governing equations used to model the FSI problem and a review of the CFEI formulation has been presented. Section 3 presents the scheme with second-order accuracy in time. The stability of the scheme is proved in Section 4. In Section 5, we present the convergence and verification results. Section 6 presents the application of CFEI formulation to study the flapping dynamics of a thin structure in a uniform axial flow. The major conclusions of this work are reported in Section 7.

2. Combined field with explicit advancing

Before the presentation of our second-order scheme, we provide for completeness a short description of the fluid–structure system and the combined field formulation. The governing equations for the fluid are written in an ALE form while the structural equation is formulated in a Lagrangian way.

2.1. Fluid–structure equations

Let $\Omega^{\mathrm{f}}(t) \subset \mathbb{R}^{\mathrm{d}}$ be a fluid domain at time t, where d is the space dimension. The motion of an incompressible viscous fluid in $\Omega^{\mathrm{f}}(t)$ is governed by the following Navier–Stokes equations: