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On the effective accuracy of spectral-like optimized finite-difference schemes for computational aeroacoustics

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ABSTRACT

The present article aims at highlighting the strengths and weaknesses of the so-called spectral-like optimized (explicit central) finite-difference schemes, when the latter are used for numerically approximating spatial derivatives in aeroacoustics evolution problems. With that view, we first remind how differential operators can be approximated using explicit central finite-difference schemes. The possible spectral-like optimization of the latter is then discussed, the advantages and drawbacks of such an optimization being theoretically studied, before they are numerically quantified. For doing so, two popular spectral-like optimized schemes are assessed via a direct comparison against their standard counterparts, such a comparative exercise being conducted for several academic test cases. At the end, general conclusions are drawn, which allows us discussing the way spectral-like optimized schemes shall be preferred (or not) to standard ones, when it comes to simulate real-life aeroacoustics problems.

1. Introduction

Highly-accurate numerical schemes for Computational AeroAcoustics (CAA) have seen widespread development over the past years. More particularly, the need for simulating acoustic waves over long propagation distances and time durations implies that numerical schemes of minimal dissipation and dispersion properties are used. Indeed, unlike for aerodynamic simulations, CAA calculations cannot stand dissipation nor dispersion errors, which may lead to a complete degradation of the signals to be simulated [1].

Consequently, regarding the use for aeroacoustic applications, high-order Finite-Difference (FD) methods have emerged as a valuable alternative to more traditional low-order methods, given their higher accuracy and, thus, ability to simulate wave-like signals on coarser grids [2]. Nevertheless, using such high-order schemes results in less efficient calculations since, for a given error tolerance, they cost more in terms of operations per node than lower-order schemes [3].

With the view of relaxing the latter constraint, one can seek to optimize FD schemes so that their error level remains low enough, whereas their numerical cost is kept reasonable [4]. In particular, regarding space derivative operators, several authors proposed optimizing FD schemes following a spectral strategy, which objective is to allot them with better dispersive properties. As an example of such spectral-like optimized FD operators, one can here recall the implicit compact [4–8], the ENO [9], the Dispersion-Relation-Preserving (DRP) [10] and the Bogey and Bailly's [11] schemes.

For each one of the referred schemes, the corresponding optimization procedure is carried out in the wavenumber space. Indeed, this so-called *spectral-like* optimization seeks at enhancing the scheme's ability to preserve the acoustic dispersion properties, by minimizing the error it induces on the spectral components of any given harmonic signal. For achieving such

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a minimization, however, one or several of the conditions ensuring the scheme to reach its maximal order-of-accuracy (i.e. a low level of truncation error) are to be dropped. As will be demonstrated later, the consequences of such an accuracy order reduction can restrict the effective range of applicability of spectral-like optimized schemes.

With the view of characterizing at best the spectral-like optimization approach, a dedicated work was thus recently achieved, which constitutes the matter of the present article. First, a detailed analysis of the FD derivative operator is conducted, along with a characterization of its associated error. Then, the main strengths and weaknesses associated with spectral-like optimization are theoretically studied, before they are numerically quantified for two of the most popular spectral-like optimized schemes, this being achieved via a direct comparison with their standard counterparts. Such an assessment is conducted with the help of various academic test cases, which allow drawing clear trends about how the schemes addressed here may behave, especially when applied to real-life problems. At the end, general conclusions are drawn.

2. Discrete approximation of differential operators via explicit finite-difference schemes

2.1. Theoretical background

Let us consider the exact mth-order partial differential operator $\partial_x^m[.]$. For a given 1D stencil of uniform discretization step Δx, we can approximate this exact differentiation with the help of a discrete operator, $\tilde{\partial}_x^m[.]$, to be based on explicit FD schemes:

$$\tilde{\partial}_x^m[.] = \frac{1}{(\Delta x)^m} \sum_{j=-M}^{N} b_{j,m} \tau_{(j\Delta x)}[.]. \tag{1}$$

In the above, $b_{j,m}$ indicate the scheme's characteristic coefficients, which values are still to be determined. Additionally, τ is the translation operator that, for any continuous quantity $f:\mathbb{R} \longrightarrow \mathbb{R}$ and a given point $x \in \mathbb{R}$, gives:

$$\tau_h[f](x) = f(x+h), \quad h \in \mathbb{R}. \tag{2}$$

At this stage, one can notice that centered (resp. noncentered) schemes are such that $N = M$ (resp. $N \neq M$). One can also remark that, for $m = 1$, (1) is associated with the well-known FD first derivative in space (with $b_{j,1}$ coefficients usually indicated as a_j). On another hand, for $m = 0$, the same equation is this time associated with the FD filter in space (with $b_{j,0}$ coefficients usually indicated as f_j).

2.2. Error operator

The formal error operator resulting from the above approximation of the exact mth-order differential operator is defined as:

$$\epsilon_m[.] = \partial_x^m[.] - \tilde{\partial}_x^m[.]. \tag{3}$$

By inserting (1) in (3), and by considering the Taylor expansion of the translation operator:

$$\tau_{(j\Delta x)}[.] = \sum_{p=0}^{+\infty} \frac{(j\Delta x)^p}{p!} \partial_x^p[.], \tag{4}$$

one gets

$$\epsilon_m[.] = \frac{1}{(\Delta x)^m} \sum_{p=0}^{+\infty} (\delta_{m,p} - \chi_p) \Delta x^p \partial_x^p[.], \quad \text{with } \chi_p = \frac{1}{p!} \sum_{j=-M}^{N} b_{j,m} j^p. \tag{5}$$

In the above, $\delta_{m,p}$ denotes the Kronecker symbol, which is of nonzero unitary value if and only if $p = m$.

When applied to a given function, $f:\mathbb{R} \longrightarrow \mathbb{R}$, the operator $\tilde{\partial}_x^m[.]$ induces a specific error, which results from both the formal error operator *and the function itself*:

$$\epsilon_m[f](x) = \frac{1}{(\Delta x)^m} \sum_{p=0}^{+\infty} (\delta_{m,p} - \chi_p) \Delta x^p \partial_x^p[f](x). \tag{6}$$

Following (6), the discrete differential operator $\tilde{\partial}_x^m[.]$ is said to be Pth-order accurate if $\epsilon_m[f](x) = O(\Delta x^P)$ when $\Delta x \rightarrow 0$, which occurs when the following $P + m$ relations are verified simultaneously:

$$\chi_p = \delta_{m,p}, \quad \text{for } p = 0, 1, \ldots, P + (m-1). \tag{7}$$