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A second-order accurate in time IMplicit–EXplicit (IMEX) integration scheme for sea ice dynamics

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ABSTRACT

Current sea ice models use numerical schemes based on a splitting in time between the momentum and continuity equations. Because the ice strength is explicit when solving the momentum equation, this can create unrealistic ice stress gradients when using a large time step. As a consequence, noise develops in the numerical solution and these models can even become numerically unstable at high resolution. To resolve this issue, we have implemented an iterated IMplicit–EXplicit (IMEX) time integration method. This IMEX method was developed in the framework of an already implemented Jacobian-free Newton–Krylov solver. The basic idea of this IMEX approach is to move the explicit calculation of the sea ice thickness and concentration inside the Newton loop such that these tracers evolve during the implicit integration. To obtain second-order accuracy in time, we have also modified the explicit time integration to a second-order Runge–Kutta approach and by introducing a second-order backward difference method for the implicit integration of the momentum equation. These modifications to the code are minor and straightforward. By comparing results with a reference solution obtained with a very small time step, it is shown that the approximate solution is second-order accurate in time. The new method permits to obtain the same accuracy as the splitting in time but by using a time step that is 10 times larger. Results show that the second-order scheme is more than five times more computationally efficient than the splitting in time approach for an equivalent level of error.

1. Introduction

Various mechanisms associated with sea ice dynamics play a key role in shaping the ice cover of the polar oceans. To properly model the processes of lead and pressure ridge formation, sea ice models require a sophisticated representation of sea ice rheology, i.e. the relation between internal stresses, material properties (ice strength) and deformations of the ice cover. Most current sea ice models use the Viscous-Plastic (VP) formulation of Hibler [1] to represent these ice interactions. The VP formulation leads to a very nonlinear problem which is known to be difficult to solve.

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Almost all sea ice model time integration schemes are based on a splitting in time between the momentum and the continuity equations (e.g., [1–5]). This means that when solving the momentum equation, the thickness distribution (including the amount of open water) is held constant at the previous time level (it, however, varies spatially). Once the velocity field is obtained, the thickness distribution is advanced to the next time level. Furthermore, an operator splitting approach is generally used to separate the change of the thickness distribution associated with advection and the growth/melt related to thermodynamic processes (e.g., [2,3]). This paper focuses on dynamics and we therefore only discuss the solution of the momentum equation and of the continuity equation without the thermodynamic source terms.

Current sea ice model numerical schemes suffer from significant numerical issues. First, as explained by Lipscomb et al. [2] (see also [6]), the splitting in time approach leads to noise in the numerical solution and can even make the model numerically unstable. As an illustrative example, consider ice converging toward a coast due to an onshore wind; a stress gradient, associated with an ice strength gradient, develops to oppose the wind stress. When using a large time step with the splitting in time approach, an unrealistically large ice strength gradient can occur. The stress gradient force can then overcompensate the wind stress and cause an unrealistic reversal of the flow (the ice then diverges at the coast). This instability, fundamentally numerical, can be cured by reducing the time step. Unfortunately, this obviously increases the total computational time. Lipscomb et al. [2] proposed a modification to the ridging scheme in order to mitigate this problem. Hutchings et al. [6] introduced a strength implicit algorithm to eliminate this instability.

A second numerical issue is related to the solution of the momentum equation. The rheology term, which determines the deformations of the ice cover based on the internal ice stresses, causes the momentum equation to be very nonlinear. Indeed, the VP rheology leads to a large change in the internal stresses when going from a slightly convergent flow to a slightly divergent one (same idea for shear stresses). The current numerical solvers for the momentum equation, however, have difficulties in finding the solution of this very nonlinear problem. There are two main classes of schemes to solve the momentum equation: the implicit solvers, which involve an outer loop iteration (sometimes referred to as Picard iteration, [5,7,8]) and the ones based on the explicit solution of the momentum equation using the Elastic–VP approach [9,10]. Both of these approaches, however, lead to a very slow convergence rate [8,10] if they converge at all [10,11]. Because of this slow convergence rate, it is typical to perform a small number of Picard iterations or of subcycling iterations. The approximate solution therefore contains residual errors which are carried on in the time integration.

To resolve this slow convergence rate issue, Lemieux et al. [4] developed a Jacobian-free Newton–Krylov (JFNK) implicit solver. They showed that the JFNK solver leads to a more accurate solution than the EVP solver [11] and that it is significantly more computationally efficient than a Picard approach [4]. Following the work of Lemieux et al. [4], Losch et al. [12] have recently developed a parallel JFNK solver for the MIT general circulation model with sea ice [13]. The numerical approaches of Lemieux et al. [4] and Losch et al. [12], however, still rely on the splitting in time scheme and are therefore susceptible to exhibit the numerical instability issue.

It is the purpose of this paper to introduce a fast and accurate time integration scheme that resolves the instability associated with the splitting in time approach. One possibility would be to solve fully implicitly the momentum and continuity equations. This avenue would imply significant modifications to the code and would be quite complex to implement. Instead, the splitting in time issue is cured by using an iterated IMplicit–EXplicit (IMEX) approach when solving the momentum and continuity equations. This approach is built around our existing JFNK solver. Basically, the idea is to move the explicit calculation of the thickness distribution inside the implicit Newton loop. We take this approach one step further by modifying the time integration in order to get second-order accuracy in time for the full system. To do so, we introduce a second-order Runge–Kutta scheme for the advection operation and discretize in time the momentum equation using a second-order backward difference (as in [14]). This paper is inspired by the strength implicit scheme of [6] and by the work of [15,16] on an iterated IMEX method for radiation hydrodynamics problems.

The main contribution of this paper is the development and demonstration of a first-of-a-kind second-order accurate in time iterated IMEX integration scheme for sea ice dynamics. This manuscript also shows the gain in accuracy and computational time of the second-order IMEX method compared to the common first-order integration scheme based on the splitting in time.

It is worth mentioning that some authors have recently questioned the validity of the VP rheology. Sea ice models based on a VP rheology do not capture the largest deformations events [17] and statistics of simulated deformations do no match observations [17] in both space and time [18]. While some authors propose new and very different formulations of ice interactions [19,20], others claim that a VP rheology with modified yield curve and flow rule can adequately represent the sea ice deformations [21]. These new physical parameterizations, under evaluation, also lead to very nonlinear problems which would also clearly benefit from the availability of reliable and efficient numerical schemes.

This paper is structured as follows. Section 2 describes the sea ice momentum equation with a VP formulation and the continuity equation. In Section 3, the discretization of the momentum and continuity equations and the descriptions of the standard splitting in time and new IMEX integration schemes are presented. In Section 4, more information about the model is given. The description of the experiments and the results are outlined in Section 5. A discussion and concluding remarks are provided in Section 6.