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Dynamic implicit 3D adaptive mesh refinement for non-equilibrium radiation diffusion ☆

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ABSTRACT

The time dependent non-equilibrium radiation diffusion equations are important for solving the transport of energy through radiation in optically thick regimes and find applications in several fields including astrophysics and inertial confinement fusion. The associated initial boundary value problems that are encountered often exhibit a wide range of scales in space and time and are extremely challenging to solve. To efficiently and accurately simulate these systems we describe our research on combining techniques that will also find use more broadly for long term time integration of nonlinear multi-physics systems: implicit time integration for efficient long term time integration of stiff multi-physics systems, local control theory based step size control to minimize the required global number of time steps while controlling accuracy, dynamic 3D adaptive mesh refinement (AMR) to minimize memory and computational costs, Jacobian Free Newton–Krylov methods on AMR grids for efficient nonlinear solution, and optimal multilevel preconditioner components that provide level independent solver convergence.

1. Introduction

In the fields of astrophysics and inertial confinement fusion the time dependent non-equilibrium radiation diffusion equations are important for solving the transport of energy through radiation in an optically thick regime. In this paper we employ a form of the model that has a flux-limited diffusion approximation (gray approximation) for the energy density coupled to a material temperature equation that incorporates a nonlinear material conduction term [1–5]. This nonlinear, coupled, time dependent set of partial differential equations (PDEs) exhibits multiple temporal and spatial scales, and the associated initial boundary value problems are highly stiff and challenging to solve. As a result, they are also an excellent testbed for the development of simulation methods for long term time integration of stiff multi-physics systems.

In this paper we will limit our scope to fully implicit time integration methods. This then enables the use of timestep control methods based on accuracy considerations and enables us to leverage the theoretical advances for accuracy based timestep control that exist in the field of ordinary differential equations (ODEs). We experiment with different adaptive

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timestep control methods including control theory based approaches that attempt to monitor and control the temporal accuracy at each timestep and minimize the total number of timesteps required over the course of the simulation. The use of control theoretic approaches to timestep control is new for radiation-diffusion calculations and is only beginning to be used for multi-physics calculations. Variable step fully implicit time integration is combined with 3D dynamic structured adaptive mesh refinement (AMR) [6] with an objective towards minimizing the total number of degrees of freedom required over the course of a simulation. Care is however required in combining these techniques as spatial regridding during dynamic AMR can introduce non-stiff transient errors that significantly affect the behavior of timestep control algorithms and can lead to a dramatic increase in the total number of timesteps required over uniform spatial grid calculations when not properly controlled. We will report on our experiences with different timestep controllers and the modifications required in this context for AMR as the literature on this topic, particularly for multi-physics simulations is sparse. Fully implicit time integration methods require highly efficient solution of the nonlinear systems at each timestep in order to be competitive with other methods. Here, we choose to use Jacobian Free Newton–Krylov (JFNK) methods with physics based preconditioning. JFNK methods allow us to avoid the formation of the full Jacobian matrices across AMR grid hierarchies which can be problematic and programming intensive for flux based finite volume discretizations on 3D AMR grid hierarchies which incorporate coarse-fine interpolation across grid levels. JFNK methods often obtain their efficiency from careful design of preconditioners. Efficient preconditioners on uniform grids for JFNK methods often employ multigrid solvers to tackle elliptic components to deliver grid independent performance. In the context of AMR, particularly for problems with elliptic components, preconditioner performance can degrade as the number of refinement levels in the AMR hierarchy increases if proper care is not paid to coupling between levels. By employing suitable multilevel preconditioner components we will demonstrate level independent performance of our nonlinear solvers for non-equilibrium radiation diffusion applications.

The remainder of this paper is organized as follows. Section 2 of this paper surveys related work in the context of equilibrium and non-equilibrium radiation diffusion problems. Section 3 describes the model problem and its temporal and spatial discretization. Section 4 describes the JFNK method and the multilevel preconditioners employed. Section 5 presents numerical results and Section 6 presents conclusions and directions for future and ongoing work.

2. Related work

In [7], Rider, Knoll and Olson introduced the idea of physics based preconditioning in 1D for non-equilibrium radiation diffusion problems. Further work by Mousseau, Knoll, Rider [4] and Mousseau, Knoll [5] extended this methodology to problems in 2D on uniform grids. Their work related to physics-based preconditioning will be leveraged here with major extensions for 3D AMR grids and multilevel preconditioners. In [3], Mavriplis compared two different approaches to solving the nonlinear systems at each timestep by considering Newton–Multigrid and Full Approximation Scheme (FAS) using agglomeration ideas on unstructured grids for this problem. In [2] Olson considers the use of efficient operator split time integration schemes on uniform grids. Work by Lowrie et al. [8] compares different time integration methods for non-equilibrium radiation diffusion while Brown, Shumaker, Woodward [9] focus on fully implicit methods and high order time integration on uniform grids. The motivation to consider automatic timestep control in our work was partially derived from [9]. We build on their work to further consider the use of the control theory based timestep controllers that provide computational stability as described in [10] and related references and consider modifications that are required for AMR. Glowinski and Toivanen [11] consider using automatic differentiation and system multigrid. Shestakov, Greenough, and Howell [12] consider pseudo-transient continuation on AMR grids using an alternative formulation. Also worth mentioning is related work for *equilibrium* radiation diffusion problems. Stals [13] compares the performance of Newton–Multigrid and FAS with local refinement on *unstructured* grids and Pernice, Philip [14] use JFNK with a Fast Adaptive Composite Grid (FAC) preconditioner on AMR grids for single physics equilibrium radiation-diffusion on structured adaptive mesh refinement (SAMR) grids.

3. Problem formulation and discretization

3.1. Model problem

The non-dimensional model equations considered in this paper are given by [1–5]:

$$\frac{\partial E}{\partial t} - \nabla \cdot (D_E \nabla E) = \sigma_a\big(T^4 - E\big) \quad \text{in } \Omega, \tag{1}$$

$$\frac{\partial T}{\partial t} - \nabla \cdot (D_T \nabla T) = -\sigma_a\big(T^4 - E\big) \quad \text{in } \Omega, \tag{2}$$

where E is the radiation energy density, T the material temperature, ∇ the gradient, $\nabla\cdot$ the divergence operator, D_E and D_T are nonlinear diffusion coefficients given by

$$D_E = \frac{1}{3\sigma_a},$$

$$D_T = kT^{\frac{5}{2}},$$