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Discontinuous Galerkin discretization of the Reynolds-averaged Navier–Stokes equations with the shear-stress transport model

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ABSTRACT

In this article we consider the development of Discontinuous Galerkin (DG) methods for the numerical approximation of the Reynolds-averaged Navier–Stokes (RANS) equations with the shear-stress transport (SST) model by Menter. This turbulence model is based on a blending of the Wilcox k–ω model used near the wall and the k–ϵ model used in the rest of the domain where the blending functions depend on the distance to the nearest wall. For the computation of the distance of each quadrature point in the domain to the nearest of the curved, piecewise polynomial wall boundaries, we propose a stabilized continuous finite element (FE) discretization of the eikonal equation. Furthermore, we propose a new wall boundary condition for the dissipation rate ω based on the projection of the analytic near-wall behavior of ω onto the discrete ansatz space of the DG discretization. Finally, we introduce an artificial viscosity to the discretization of the turbulence kinetic energy (k-)equation to suppress oscillations of k near the underresolved boundary layer edge.
The wall distance computation based on the continuous FE discretization of the eikonal equation is demonstrated for an internal and three external/aerodynamic flow geometries including a three-element high-lift configuration. The DG discretization of the RANS equations with the SST model is demonstrated for turbulent flows past a flat plate and the RAE2822 airfoil (Cases 9 and 10). The results are compared to the underlying k–ω model and experimental data.

1. Introduction

In the past years there has been a tremendous progress in the development of higher-order Discontinuous Galerkin (DG) methods for turbulent flows as governed by the Reynolds-averaged Navier–Stokes (RANS) equations [1–11]. These developments concentrated on DG discretizations of the RANS equations with basically two different turbulence models: The one-equation Spalart–Allmaras model [12] and the two-equation Wilcox k–ω model [13,14]. The two-equation k–ω model, which was designed for Finite Volume (FV) methods by Wilcox [13], is mostly used unchanged for DG methods. Only some minor modifications have been introduced to stabilize the methods such as a $\log(\omega)$-transformation and a realizability condition on ω to avoid unphysical states [2]. Still, the computation of higher-order DG solutions to the RANS equations with k–ω based turbulence models remains a challenge.

Like Menter [15,16] noticed for 2nd order FV methods also for higher-order methods the flow solution has a strong dependence on the freestream values of ω. To avoid this Menter combined the classical k–ω model with the k–ϵ model in a k–ω formulation. The idea is to use k–ω near the wall to profit from the good near-wall behavior of this model and use

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the k–ϵ model from the middle of the boundary layer up to the farfield to reduce the dependence on freestream values. The switch between both models is realized based on blending functions, which depend among others on the distance to the nearest wall boundary. The resulting model is the so-called baseline (BSL) model by Menter [17,18]. Menter also noticed that in adverse pressure gradient flows the eddy-viscosity has to be modified. This modification in combination with the BSL model results in the so-called shear-stress transport (SST) model [19]. The implementation of the two models is in principle the same as for the standard k–ω model. One of the differences is their dependence on the wall distance.

Depending on the complexity of the geometry already for finite volume methods the computation of the nearest wall distances of all cell centers or vertices becomes an expensive and in parallel computations a rather complicated task [20]. In Discontinuous Galerkin methods this computational complexity is significantly increased due to several reasons. First, the distance to the nearest wall boundary is required for all cell and face quadrature points, e.g. 125 Gaussian cell quadrature points and $6 \times 25 = 150$ Gaussian face quadrature points for $p = 3$ polynomials on a hexahedral element. Second, in DG methods curved wall boundaries must be approximated by piecewise higher-order polynomials [21]. Thus the distance of each quadrature point to a possibly large number of higher-order polynomial wall boundary faces is required. And finally, following the arguments of [20] the approximation of the wall distance function should be smooth. In view of its usage in a higher-order DG discretization it would even be desirable that the approximation of the wall boundary function is of higher order.

In this work, we approximate the wall boundary function by a higher-order finite element (FE) discretization of the eikonal equation [20]. The underlying FE discretization was first introduced by Liu et al. [22] and applied to internal flow problems. Not being bound by walls like in internal flows, it turned out, however, that the discretization in [22] is not stable on the computational domains of the external flow problems typically considered in aerodynamics. Therefore, we stabilize the FE discretization using the streamline-diffusion method. For more complex geometries like high-lift configurations a further stabilization of the discretization is required based on artificial viscosity. In Section 4 we give details of the stabilized FE discretization of the eikonal equation and present some numerical results for an internal and an external flow problem as well as for the L1T2 high-lift configuration [23].

Another important ingredient in the discretization of k–ω based turbulence models like the BSL or SST model is the discretization of the wall boundary condition. The turbulent kinetic energy k has a theoretical value of zero at the wall, but the dissipation rate ω becomes infinity. Based on the analytic solution behavior of ω near the wall boundary Menter [17] proposed the value $\omega_w = 10\frac{6\mu}{\beta y_1^2}$ at the wall boundary. This "approximation" of the theoretically infinite value of ω at the boundary is based on the distance y_1 of the nearest grid point off the wall boundary, which corresponds to the first grid layer spacing in a structured mesh or in the prism layer of a hybrid mesh typically used in aerodynamics. Whereas this ω_w value proved to be sufficient for 2nd order FV methods on fine meshes, this value seems to be unappropriate in case of higher-order methods on relatively coarse meshes. Due to this, already Bassi et al. [24] proposed an ω wall boundary condition depending on the order of the polynomial approximation.

In Section 3.2 we derive a new wall boundary condition based on the projection of the near-wall behavior of ω onto the polynomial ansatz space of degree p used in the DG discretization of the RANS-$k\omega$ equations. This results in a p-dependent value of ω_w which increases for increasing p giving an improved "approximation" of the theoretically infinite value of ω at the wall under p-refinement/enrichment. On the coarse meshes typically used in higher-order methods the first layer grid spacings y_1 are very large and thus the resulting ω_w values too small. In order to avoid this, we base the ω_w value on a modified first grid layer spacing $\tilde{y}_1$. Being computed from the condition $y^+ = \tilde{y}_1 \frac{u_\tau}{\nu_w} = \mathcal{O}(1)$, where $u_\tau = \sqrt{\tau_w/\rho}$ is the friction velocity, τ_w the shear stress, and y^+ being in the range of one, the resulting $\tilde{y}_1$ is not directly dependent on the actual first grid layer spacing any more.

Some further issues need to be treated in the context of higher-order methods for k–ω based turbulence models. After reaching its maximum within the boundary layer the turbulence kinetic energy k decreases rapidly until the edge of the boundary layer where it reaches a small but positive value. Given that aerodynamic meshes are usually graded in the boundary layer, with the highest normal resolution near the wall boundary and a lower resolution off the wall, the boundary layer edge region is often underresolved, in particular on the coarse meshes typically used for higher-order flow computations. In this underresolved boundary layer edge region the k-profile has a very rapid change in its slope leading to an undershoot and numerical oscillations. This might even result in negative (and thus unphysical) values of k. The blending functions in Menter's BSL and SST models are very sensitive to changes in k. The unphysical values of k lead to unwanted effects in the blending functions. The computation of the square-root of k is only one of the problems. This kind of oscillations near the boundary layer edge have already been seen in Nguyen et al. [7] in the working variable $\tilde{\nu}$ of the Spalart–Allmaras (SA) one-equation turbulence model. [7] cured this effect by introducing artificial viscosity. Also we stabilize the k-equation with an artificial viscosity term in order to smooth the k-profile, i.e. avoid the undershoot and reduce the oscillations in k. In Section 3.1 we give details of the proposed k-smoothing viscosity term and its effect on the behavior of k in the boundary layer. Note that an effect of this artificial viscosity is visible only close to the boundary layer edge. Furthermore, the artificial viscosity vanishes as the resolution of the discretization increases, i.e. under mesh refinement or an increase of the polynomial degree.

The paper is structured as follows. After introducing, in Section 2, the standard k–ω turbulence model, the BSL and the SST model, we give some details on the DG discretization including the k-smoothing artificial viscosity term and the wall boundary condition in Section 3. In Section 4 we introduce the FE discretization of the eikonal equation and demonstrate it