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An immersed boundary method for two-fluid mixtures

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ABSTRACT

We present an immersed boundary method for interactions between elastic boundaries and mixtures of two fluids. Each fluid has its own velocity field and volume-fraction. A penalty method is used to enforce the condition that both fluids' velocities agree with that of the elastic boundaries. The method is applied to several problems: Taylor's swimming sheet problem for a mixture of two viscous fluids, peristaltic pumping of a mixture of two viscous fluids, with and without immersed particles, and peristaltic pumping of a mixture of a viscous fluid and a viscoelastic fluid. The swimming sheet and peristalsis problems have received much attention recently in the context of a single viscoelastic fluid. Numerical results demonstrate that the method converges and show its capability to handle a number of flow problems of substantial current interest. They illustrate that for each of these problems, the relative motion between the two fluids changes the observed behaviors profoundly compared to the single fluid case.

1. Introduction

Interactions between elastic structures and a surrounding fluid medium are ubiquitous in biological processes, from the locomotion of E. coli in intestinal fluid to the swimming of sperm through cervical mucus. A powerful computational approach for handling this type of dynamic fluid structure interaction, in the case of single, incompressible fluids, is the Immersed Boundary (IB) method originally developed by Peskin [1]. The robustness of the IB method has led to its application in many different biofluid applications. For example, it has been used to study cardiac dynamics [2–4], platelet aggregation [5,6], aquatic locomotion by small [7,8] and large [9] organisms, cellular locomotion [10], peristalsis [11,12] insect flight [13,14], biofilms [15] and feeding by marine invertebrates [16–18]. It has also been used in a wide-range of engineering applications including, for example, electrohydrodynamics [19,20] and reaction diffusion systems [21,22]. In all of this work, the fluid environment is treated as a single continuous medium. However, many biological fluids such as mucus as well as many engineering and industrial fluids are mixtures of a solvent and a polymer network. There may be relative motion between the different components of the mixture and then describing the material as a single continuous medium is inappropriate. The two-fluid model is an often-used approach to describe gel mechanics, where both network and solvent coexist at each point of space, and each phase is modeled as a continuum with its own velocity field and constitutive law. Recently, analytical studies of G.I. Taylor's classical swimming-sheet problem with two-fluid models have yielded interesting and unexpected results [23]. The analytical studies have been limited to simple geometries and small amplitude motions. In this paper, we present an extension of the classical immersed boundary method to a mixture of two fluids, in which both

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fluids satisfy the no-slip condition on the immersed structures. Enforcing this condition is one of the challenges in a two-fluid IB method. In the classic single-fluid IB method, the no-slip condition between the immersed boundary and the fluid is used to define the velocity of each point on the immersed structure as being the same as that of the adjacent fluid. In developing a two-fluid IB method, the question that arises is at which fluid's velocity (or some average of them) points on the immersed boundary should move. Unless the two fluids have the same velocity at each point of the immersed structure, any of these choices for the velocity of the IB point is problematic and allows one or both fluids to move relative to the immersed structure. The key idea in our two-fluid IB method is to introduce two sets of immersed boundaries to represent the same elastic structure. Penalty forces are introduced to keep the boundaries moving at approximately the same velocity.

We apply the new method to three problems that illustrate its capabilities and demonstrate that accounting for relative motion between the two fluids can profoundly change the overall motion compared to the case of a single fluid. The first problem is that of determining the swimming speed of the classical Taylor's undulating sheet in a mixture of two viscous fluids. For this problem, the motion of the undulating sheet is (approximately) imposed and the resulting fluid velocities, pressure, and volume-fractions are determined. As described in detail in [31], our simulations show that for two viscous fluids with different viscosities, the swimming speed is always less than for a single viscous fluid. Here we show the computed velocity fields and volume-fraction evolution, and we use this problem to numerically assess the convergence of the new method. For the second problem, we look at peristaltically-driven motion of a mixture of two viscous fluids. We see that the total flux driven by the peristalsis is greater for the mixture of fluids by an amount that depends on the ratio of the viscosities and on the composition (volume fractions) of the mixture. The flux is further altered by the presence of suspended elastic-walled particles. We also see two new aspects of the method's performance: (i) the motion of the fluid (and suspended particles) within the tube is not sensitive to the composition of the fluid mixture outside of the tube, and (ii) the penalty forces are effective in ensuring that the network and solvent velocities are equal on the surfaces of the freely-moving suspended particles. For the third problem, we again look at peristalsis, but this time the network is viscoelastic and has time- and space-varying material properties that change as the system evolves. In this example, we see that network viscoelasticity and the possibility of relative motion between the solvent and the network have opposing effects on the net flux driven by the peristaltic motion.

The paper is organized as follows. In Section 2, we introduce the model equations and the numerical method. In Sections 3.1, 3.2, and 3.3, we present, respectively, numerical simulations of Taylor's swimming sheet problem for a mixture of two viscous fluids, simulations of peristaltic pumping of a mixture of two viscous fluids (both with and without immersed elastic particles), and simulations of peristaltic pumping of a fluid mixture composed of a viscoelastic network and a viscous solvent. Finally, in Section 4 we offer concluding remarks.

2. Model equations and numerical method

Our development of a two-material immersed boundary method draws on our previous work on numerical methods for two-material mixtures [24] and on the immersed boundary methodology for a single fluid [1], so we begin this section by introducing the two-material model and reviewing the classical immersed boundary method.

2.1. Two-material mixture model

We consider a mixture of two materials, which we denote "solvent" and "network", in which both materials may be present at any spatial point $\mathbf{x}$. While for some applications in this paper, the two materials are treated as immiscible viscous fluids, we use the solvent/network terminology throughout the paper for simplicity. The relative amounts of the two materials are given by the volume-fractions, $\theta^{\mathrm{s}}(\mathbf{x},t)$ and $\theta^{\mathrm{n}}(\mathbf{x},t)$ for the solvent and network, respectively. We assume that the total amount of each material remains constant. The two volume-fractions evolve according to the continuity equations

$$\frac{\partial\theta^{\mathrm{n}}}{\partial t}+\nabla\cdot\left(\theta^{\mathrm{n}}\mathbf{u}^{\mathrm{n}}\right)=0, \tag{1}$$

$$\frac{\partial\theta^{\mathrm{s}}}{\partial t}+\nabla\cdot\left(\theta^{\mathrm{s}}\mathbf{u}^{\mathrm{s}}\right)=0, \tag{2}$$

where $0<\theta^{\mathrm{n}}<1$, $\theta^{\mathrm{s}}=1-\theta^{\mathrm{n}}$ and $\mathbf{u}^{\mathrm{n}}(\mathbf{x},t)$ and $\mathbf{u}^{\mathrm{s}}(\mathbf{x},t)$ are the network and solvent velocity respectively. Since $\theta^{\mathrm{s}}+\theta^{\mathrm{n}}=1$, adding these two equations gives an incompressibility constraint

$$\nabla\cdot\left(\theta^{\mathrm{n}}\mathbf{u}^{\mathrm{n}}+\theta^{\mathrm{s}}\mathbf{u}^{\mathrm{s}}\right)=0 \tag{3}$$

on the volume-fraction averaged velocity $\theta^{\mathrm{n}}\mathbf{u}^{\mathrm{n}}+\theta^{\mathrm{s}}\mathbf{u}^{\mathrm{s}}$.

The two velocity fields are determined from the momentum equations for the two materials. For this paper, we restrict our attention to the viscous-dominated situation in which inertial terms are ignored (similar to zero-Reynolds number flow) and in which the velocities and pressure respond instantaneously to applied forces. Under these conditions, the momentum equations reduce to the force-balance equations

$$\nabla\cdot\left(\theta^{\mathrm{n}}\tau\right)+\nabla\cdot\left(\theta^{\mathrm{n}}\boldsymbol{\sigma}^{\mathrm{n}}\right)-\xi\theta^{\mathrm{n}}\theta^{\mathrm{s}}\left(\mathbf{u}^{\mathrm{n}}-\mathbf{u}^{\mathrm{s}}\right)-\theta^{\mathrm{n}}\nabla p+\mathbf{f}^{\mathrm{n}}=0, \tag{4}$$

$$\nabla\cdot\left(\theta^{\mathrm{s}}\boldsymbol{\sigma}^{\mathrm{s}}\right)-\xi\theta^{\mathrm{n}}\theta^{\mathrm{s}}\left(\mathbf{u}^{\mathrm{s}}-\mathbf{u}^{\mathrm{n}}\right)-\theta^{\mathrm{s}}\nabla p+\mathbf{f}^{\mathrm{s}}=0. \tag{5}$$