Journal of Computational Physics 259 (2014) 284–303

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Algebraic multiscale solver for flow in heterogeneous porous media

Yixuan Wang [a,*], Hadi Hajibeygi [b,a], Hamdi A. Tchelepi [a]

[a] Department of Energy Resources Engineering, Stanford University, 367 Panama St., Rm. 065, Stanford, CA 94305-2220, USA
[b] Department of Geoscience and Engineering, Faculty of Civil Engineering and Geosciences, Delft University of Technology, P.O. Box 5048, 2600 GA Delft, The Netherlands

ARTICLE INFO

Article history:
Received 6 June 2013
Received in revised form 20 November 2013
Accepted 24 November 2013
Available online 6 December 2013

Keywords:
Multiscale methods
Iterative multiscale methods
Algebraic multiscale solver
Scalable linear solvers

ABSTRACT

An Algebraic Multiscale Solver (AMS) for the pressure equations arising from incompressible flow in heterogeneous porous media is described. In addition to the fine-scale system of equations, AMS requires information about the superimposed multiscale (dual and primal) coarse grids. AMS employs a global solver only at the coarse scale and allows for several types of local preconditioners at the fine scale. The convergence properties of AMS are studied for various combinations of global and local stages. These include MultiScale Finite-Element (MSFE) and MultiScale Finite-Volume (MSFV) methods as the global stage, and Correction Functions (CF), Block Incomplete Lower–Upper factorization (BILU), and ILU as local stages. The performance of the different preconditioning options is analyzed for a wide range of challenging test cases. The best overall performance is obtained by combining MSFE and ILU as the global and local preconditioners, respectively, followed by MSFV to ensure local mass conservation. Comparison between AMS and a widely used Algebraic MultiGrid (AMG) solver [1] indicates that AMS is quite efficient. A very important advantage of AMS is that a conservative fine-scale velocity can be constructed after any MSFV stage.

1. Introduction

Numerical simulation of multiphase flow in large-scale heterogeneous reservoirs is computationally demanding. To reduce the computational complexity, several MultiScale (MS) methods have been developed [2–10]. In MS methods, the global fine-scale problem is decomposed into large numbers of local problems. Basis functions, which are numerical solutions of local problems, are used to construct accurate coarse-scale quantities. Once the coarse-scale system is solved, the solution is mapped onto the fine scale using the basis functions. Among the proposed multiscale methods, the Mixed MultiScale Finite-Element (MMSFE) [6,11,5,8] and the MultiScale Finite Volume (MSFV) [7] methods provide locally mass-conservative solutions, which is a crucial property for solving coupled flow and transport problems.

The MSFV method employs locally computed basis functions to construct the coarse-scale system in a finite-volume framework. To obtain a locally conservative velocity field at the fine scale, additional local Neumann problems are constructed over the primal coarse control volumes. Recent developments of the MSFV method include incorporating the effects of compressibility [12,13], gravity and capillary [14], complex wells [15,16], faults [17], fractures [18], three-phase [19] and compositional displacements [20]. Furthermore, the efficiency of the method has been enhanced by adaptive computation of the basis functions for multiphase, time-dependent displacement problems [21–24].

* Corresponding author.
E-mail addresses: yixuanw@stanford.edu (Y. Wang), h.hajibeygi@tudelft.nl (H. Hajibeygi), tchelepi@stanford.edu (H.A. Tchelepi).

0021-9991/$ – see front matter
http://dx.doi.org/10.1016/j.jcp.2013.11.024

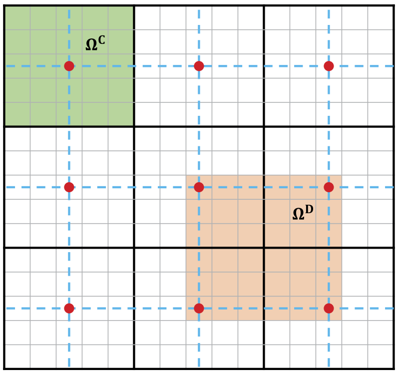

Fig. 1. Primal (bold black) and dual (dashed blue) coarse cells. Fine-cells belonging to a coarse cell (control volume) are shown in green. Fine-cells that belong to a dual coarse cell are highlighted in light orange. The red circles denote the coarse nodes (vertices). (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

For a wide range of heterogeneous test cases, the MSFV results are shown to be in good agreement with reference fine-scale solutions. However, the accuracy of MSFV method suffers from the presence of extreme permeability contrasts (e.g., SPE 10 bottom section [25]) or highly anisotropic problems (e.g., large grid aspect ratios) [26]. To overcome these limitations, the iterative MSFV (i-MSFV) method was introduced [27], where the MSFV errors were systematically reduced with the help of locally computed Correction Functions (CF). The i-MSFV had weak convergence performance for highly heterogeneous and anisotropic problems [28]. The convergence rate was improved significantly by using the MSFE operator (i-MSFE) [29,30]. The benefit of the MSFV operator is that a mass-conservative solution is obtained. Thus, MSFV can be employed at the end of the iterative process to ensure that the approximate solution is conservative.

Both the original (single-pass) and iterative multiscale methods can be formulated in an algebraic manner [13,30]. The algebraic formulation reduces the implementation complexity, especially for problems defined on unstructured grids, and it allows for easy integration of the method into existing reservoir simulators. The Two-Stage Algebraic Multiscale Solver (TAMS) [30] consists of local and global stages. In the global stage, low frequency errors are resolved by a multiscale preconditioner. In the local stage, high frequency errors are resolved using a Block ILU with zero fill-in (BILU) [31] local solver. However, CF was not incorporated into TAMS, and the exact role of CF in the context of multi-stage preconditioning had not been analyzed. In addition, the best choices among the variety of possible local and global stages have not been thoroughly investigated.

In this paper, a general iterative Algebraic Multiscale Solver (AMS) is described. AMS allows for MSFV, or MSFE, as global operators with different types of local boundary conditions, and it allows for many local fine-scale solvers, e.g., BILU and Line-Relaxation (LR) [32]. We show that the CF is an independent local preconditioning stage aimed at resolving high-frequency errors. The effects of the CF local stage on the AMS convergence rate and the overall computational efficiency for several heterogeneous problems are analyzed. To obtain the best combination of methods, we provide systematic performance tests considering different global (MSFV and MSFE) and local (BILU, CF, ILU) stages with different local boundary conditions. Then, the computational efficiency of AMS is compared with an advanced algebraic multigrid solver, SAMG, developed at Fraunhofer SCAI [1].

The paper is organized as follows. First, a general Algebraic Multiscale Solver (AMS) for heterogeneous elliptic problems is developed. Second, the effects of CF are analyzed. Then, numerical results are presented. The conclusions are given in the final section.

2. Algebraic Multiscale Solver (AMS)

In this section, the original MSFV method is reviewed briefly. Then, the Algebraic Multiscale Solver (AMS) is described.

2.1. MultiScale Finite Volume (MSFV) method

The pressure equation for single-phase incompressible flow can be written as

$$\nabla \cdot (\boldsymbol{\lambda} \cdot \nabla p) = \nabla \cdot (\rho g \boldsymbol{\lambda} \cdot \nabla z) + \tilde{q}, \tag{1}$$

where $\boldsymbol{\lambda}$ is the positive-definite mobility tensor, $\tilde{q}$ represents source terms, g is the gravitational acceleration acting in the ∇z direction, and ρ is the density.

The MSFV grid consists of two sets of overlapping coarse grids, namely primal and dual coarse grids, superimposed on the given fine grid (Fig. 1). There are N_C primal coarse cells (control volumes), Ω_i^C ($i \in \{1, \ldots, N_C\}$), and N_D dual-coarse cells (local domains), Ω_j^D ($j \in \{1, \ldots, N_D\}$).