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A mixture-energy-consistent six-equation two-phase numerical model for fluids with interfaces, cavitation and evaporation waves

Marica Pelanti [a,*], Keh-Ming Shyue [b]

[a] Department of Mechanical Engineering, École Nationale Supérieure de Techniques Avancées – ENSTA ParisTech, 828, Boulevard des Maréchaux, 91762 Palaiseau Cedex, France
[b] Department of Mathematics, National Taiwan University, Taipei 106, Taiwan

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ABSTRACT

We model liquid–gas flows with cavitation by a variant of the six-equation single-velocity two-phase model with stiff mechanical relaxation of Saurel–Petitpas–Berry (Saurel et al., 2009) [9]. In our approach we employ phasic total energy equations instead of the phasic internal energy equations of the classical six-equation system. This alternative formulation allows us to easily design a simple numerical method that ensures consistency with mixture total energy conservation at the discrete level and agreement of the relaxed pressure at equilibrium with the correct mixture equation of state. Temperature and Gibbs free energy exchange terms are included in the equations as relaxation terms to model heat and mass transfer and hence liquid–vapor transition. The algorithm uses a high-resolution wave propagation method for the numerical approximation of the homogeneous hyperbolic portion of the model. In two dimensions a fully-discretized scheme based on a hybrid HLLC/Roe Riemann solver is employed. Thermo-chemical terms are handled numerically via a stiff relaxation solver that forces thermodynamic equilibrium at liquid–vapor interfaces under metastable conditions. We present numerical results of sample tests in one and two space dimensions that show the ability of the proposed model to describe cavitation mechanisms and evaporation wave dynamics.

1. Introduction

The modeling of cavitating flows is relevant in numerous areas of engineering, from naval and submarine systems design to aerospace and nuclear power plants technologies. Cavitating fluids are multiphase mixtures that often involve complex hydrodynamic and thermodynamic processes: liquid–vapor phase transition, dynamical creation of interfaces, vapor structures collapse, and associated shock wave formation and interaction (cf. [1–3]). As a further reason of complexity, in many industrial applications these flows occur in irregular geometries and they have a multi-dimensional character.

Extensive work has been dedicated in the past decades to the simulation of cavitating flows and liquid–vapor flows with phase change, see for instance [4–15] and the references therein. Among the different modeling approaches, the class of hyperbolic compressible multiphase models stemming from the original model of Baer–Nunziato [16] has shown great capabilities in describing the complex wave patterns and thermodynamic mechanisms of cavitation. A first essential feature

* Corresponding author. Tel.: +33 1 69 31 98 19; fax: +33 1 69 31 99 97.
E-mail addresses: marica.pelanti@ensta-paristech.fr (M. Pelanti), shyue@ntu.edu.tw (K.-M. Shyue).

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of these models is that compressibility is taken into account for all phases, vapor as well as liquid. This is fundamental to correctly capture wave propagation phenomena and acoustic perturbations, and it is particularly crucial when liquid–vapor transition occurs [8]. Another important property is that these models can retain temperature and Gibbs free energy non-equilibrium effects, thus they are able to capture metastable states as well as evaporation fronts, when heat and mass transfer processes are included in the physical description through thermal and chemical relaxation source terms.

There exist various formulations of compressible temperature non-equilibrium multiphase flow models, depending on the assumptions on mechanical and kinetic phase equilibrium. In choosing a particular model, one has to find a good compromise between the accuracy of the description of the physical phenomena and the ability of conceiving robust and efficient numerical methods. In the present work, we are interested in the hyperbolic single-velocity six-equation model proposed by Saurel, Petitpas and Berry in [9] for compressible two-phase flows, see also Zein et al. [17]. This model consists of an advection equation for the volume fraction of one phase, mass and internal energy equations for each phase, and a mixture momentum equation. The six-equation model assumes instantaneous velocity equilibrium between the two phases, but it retains mechanical, thermal and chemical non-equilibrium effects. In the limit of instantaneous pressure relaxation the model reduces to the well known compressible two-phase flow model of Kapila et al. [18]. Nonetheless, as emphasized in [9], and as we briefly recall in Section 2, numerically it is more advantageous to solve the six-equation system with stiff mechanical relaxation rather than the Kapila et al. [18] pressure-equilibrium five-equation model system.

The single-velocity six-equation two-phase model with stiff pressure relaxation was employed in [9] for applications to interface problems and mechanical cavitation processes (that is cavitation with no phase transition). It was later used by Zein et al. in [17] to simulate liquid–vapor transition in metastable liquids. One difficulty of the numerical algorithm illustrated in the latter work, as noted by the authors, is that it may require a very small time step for stability for some expansion problems with phase transition, due to the stiffness of the chemical relaxation terms. Only one-dimensional numerical results are presented by the authors in [17].

The aim of the present paper is to conceive a new multiphase flow computational model on the basis of the six-equation system of [9] that could deal efficiently with interfaces, cavitation and evaporation waves, while retaining simplicity and time-affordability. The key idea of our approach is to employ an alternative mathematical formulation of the standard six-equation model system [9] in the numerical discretization. Rather than using the two phasic internal energy equations of the classical model, in our algorithm we employ two equations for the phasic total energies. Mathematically, these two model systems are equivalent. The present model, however, is numerically advantageous with respect to the standard one, since it allows us to easily design a simple numerical method that ensures important consistency properties with mixture total energy conservation and with the mixture thermodynamic state. More specifically, first, we are able to automatically recover a conservative discrete form of the mixture total energy equation, whereas the classical six-equation model system needs to be augmented with an additional conservation law for the mixture total energy to correct the thermodynamic state [9,17]. Secondly, as a consequence of the mixture total energy conservation consistency property, we are able to easily ensure agreement of the relaxed pressure at equilibrium with the correct mixture equation of state for the full six-equation two-phase model that includes mechanical and thermo-chemical stiff relaxation effects. Relaxation terms are therefore efficiently handled.

To numerically solve the proposed two-phase model with pressure, temperature, and Gibbs free energy relaxation, we employ a simple fractional step approach that consists of the homogeneous hyperbolic system solution step, and a sequence of steps thereafter to solve systems of ordinary differential equations containing the relaxation source terms. A high-resolution wave propagation method based on Riemann solvers (HLLC and Roe) (cf. [19]) is employed for the numerical solution of the homogeneous hyperbolic system. The algorithm is easily implemented in the framework of the CLAWPACK software package [20]. For solving the ordinary differential equations with stiff relaxation sources, we have devised robust solvers that drive the mixture to the desired equilibrium conditions in a sequence of relaxation processes (cf. [21,10,8,17,14]). In this procedure, similar to [8,22], thermodynamic equilibrium is forced at liquid–vapor interfaces under metastable conditions. Numerically for this task we employ an idea similar to [14,23] that uses the thermodynamic equilibrium conditions to reduce the solution of the ODEs relaxation problem to the solution of a simple system of algebraic equations for the equilibrium state variables.

This paper is organized as follows. In Section 2.1, we begin by recalling the six-equation single-velocity model with stiff mechanical relaxation of Saurel–Petitpas–Berry [9] for compressible two-phase flows. We then propose in Section 2.2 a variant of this model system, by employing phasic total energy equations in the mathematical formulation instead of the phasic internal energy equations of the classical approach. The extended model that includes thermal and chemical relaxation terms to model heat and mass transfer is described in Section 2.3. In Section 3 we illustrate the numerical method to solve the basic model system with mechanical relaxation only. In this section we also discuss the mixture-energy-consistency property of the algorithm. The numerical treatment of temperature and Gibbs free energy relaxation source terms is described in Section 4. Finally, in Section 5 we present a selection of numerical results obtained by employing the proposed method with and without activation of heat and mass transfer.