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A one-domain approach for modeling and simulation of free fluid over a porous medium

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ABSTRACT

We propose a one-domain approach based on the Brinkman model for the modeling and simulation of the transport phenomenon between free fluid and a porous medium. A thin transition layer is introduced between the free fluid region and the porous media region, across which the porosity and permeability undergo a rapid but continuous change. We study the behavior of the solution to the one-domain model analytically and numerically. Using the method of matched asymptotic expansion, we recover the Beavers–Joseph–Saffman (BJS) interface condition as the thickness of the transition layer goes to zero. We also calculate the error estimates between the leading order solution of the one-domain model and the standard Darcy–Stokes model of two-domain model with BJS condition. Numerical methods are developed for both the one-domain model and the two-domain model. Numerical results are presented to support the analytical results, thereby justifying the one-domain model as a good approximation to the two domain Stokes–Darcy model.

1. Introduction

Transport phenomena involving free fluid over porous medium have been encountered in a wide range of industrial applications (cf. [29] and the references therein). The main modeling problem lies in coupling conservation equations in both free fluid region and porous medium, which leads to the modeling of fluid–porous interface condition. Generally, there are two different formulations to deal with the problem: the so-called one-domain and two-domain approaches [8,17].

In the two-domain approach, transport equations are modeled in each homogeneous region. In the free fluid region, the flow is modeled by the Navier–Stokes equation. In the porous medium, the flow is generally represented by a macroscopic model, e.g., Darcy equation, Brinkman equation or Forchheimer equation. Darcy's law (cf. [4]) describes the linear relationship between the Darcy velocity and the gradient of pressure. Its theoretical derivation can be found in [1,18,19,44,13,39]. Brinkman [7] proposed a correction term with an effective viscosity added to Darcy's law. The Brinkman equation has been theoretically analyzed via homogenization [38,27,3] and the volume averaging method [35,3]. When the seepage velocity is high, Forchheimer [14] proposed an addition of a quadratic term of the seepage velocity into Darcy's law. Homogenization analysis and volume averaging method have also been applied to theoretically derive Forchheimer's law (cf. [36,45,11,20,2]).

For the two-domain approach, the difficulty is to determine an effective fluid/porous interface condition that connects the two different models in each domain [30]. The first attempt was made by Beavers and Joseph [5] who provided a slip boundary condition for the velocity at the interface to couple the Stokes and Darcy equations:

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$$\left.\frac{\partial u_S}{\partial y}\right|_{\Gamma^+} = \frac{\alpha}{\sqrt{K_p}}\left(u_S|_{\Gamma^+} - u_D|_{\Gamma^-}\right), \tag{1.1}$$

where K_p is the permeability of the porous medium, u_S is the free fluid velocity, u_D is the seepage Darcy velocity in the porous medium, Γ^+ and Γ^- denote the interface location on the free fluid side and the porous medium side respectively, and α is an empirical parameter. A justification of this Beavers–Joseph interface condition was given by Saffman [37] who simplified this condition by neglecting the seepage velocity u_D, and the law was presented in the form:

$$\left.\frac{\partial u_S}{\partial y}\right|_{\Gamma^+} = \frac{\alpha}{\sqrt{K_p}} u_S|_{\Gamma^+} + O(\sqrt{K_p}),$$

which has also been verified by theory of homogenization in [21–23] for the flow governed by a force coming from the pressure drop. Instead of Darcy's law used in porous medium, Neale and Nader [28] proposed an alternative approach by using the Brinkman model to obtain the Beavers-Joseph interface condition with the parameter $\alpha = \sqrt{\mu_{\text{eff}}/\mu}$, where μ is the fluid dynamic viscosity and μ_{eff} is the effective viscosity (cf. [16]) of fluid in the porous medium. A few studies have been devoted to the determination of μ_{eff} (cf. [40] and the references therein). There are some other types of interface condition being used. For instance, Ochoa-Tapia and Whitaker (OTW) [31,32] introduced the continuity of the tangential velocity but discontinuous tangential shear stress based on a non-local form of the volume averaged Stokes equation to couple the Stokes and Brinkman equations at the interface, i.e.

$$\mu \left.\frac{\partial \langle u\rangle}{\partial y}\right|_{\Gamma^+} - \mu_{\text{eff}} \left.\frac{\partial \langle u\rangle}{\partial y}\right|_{\Gamma^-} = -\frac{\beta}{\sqrt{K_p}}\langle u\rangle|_{\Gamma},$$

where $\langle u\rangle$ is the volumetric average of velocity and β is an adjustable parameter. There are some studies focusing on the determination of the parameter β (cf. [17,8,9,41–43,24,10]). When the inertia effects become important, Ochoa-Tapia and Whitaker [33] presented an improved interfacial condition as follows:

$$\frac{\mu}{\phi_p} \left.\frac{\partial \langle u\rangle}{\partial y}\right|_{\Gamma^-} - \mu \left.\frac{\partial \langle u\rangle}{\partial y}\right|_{\Gamma^+} = \beta_1 \frac{\mu}{\sqrt{K}}\langle u\rangle|_{\Gamma} + \beta_2 \rho\left(\langle u\rangle|_{\Gamma}\right)^2,$$

where ϕ_p is porosity of porous medium, ρ is mass density of fluid phase, β_1 and β_2 are adjustable parameters. Since the model considered in this paper does not encounter the inertial effects, the survey of interfacial conditions in this paper does not cover the above condition.

In the one-domain approach, a single set of transport equation is used for the entire domain consisting the porous medium and free fluid regions and the interface which is a continuous transition zone of thickness ε (cf. Fig. 1), across which the physical variables undergo strong but continuous variations [8]. The interface can also be viewed as an ideal representation of a region with continuous spatial variation of the macroscopic properties, such as permeability, porosity [17,42]. Then following this approach, an explicit function of the jump parameter β in the above OTW interface condition is obtained in [17], which is related to the continuous spatial variations in the transition region. Moreover, Chandesris and Jamet [8] gave another explicit function for β by using the method of matched asymptotic expansion (MMAE) in the transition region for one-dimensional case. In addition, one can refer to [25] for some comparisons between the two-domain approach and another type of discontinuous one-domain approach which is interpreted in the sense of distributions.

There have been many work on numerical methods for the two-domain approach. Various numerical methods, such as finite element methods, mixed methods, discontinuous Galerkin methods and combinations of these methods, have been

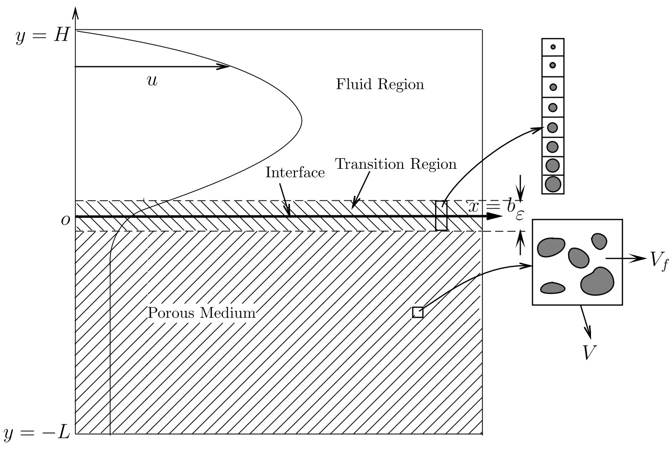

Fig. 1. Free fluid flow over porous medium.