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# Coupling of Boundary Element and Wave Based Methods for the efficient solution of complex multiple scattering problems

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ABSTRACT

A novel hybrid method for the efficient solution of complex acoustic multiple scattering problems is proposed in this paper. The Wave Based Method and the Boundary Element Method are coupled to benefit from the strengths of both. The former is an indirect Trefftz approach, which has a faster convergence rate and lower computational load compared to element based methods when applied on geometries of moderate complexity. The latter is the state-of-the-art technique for unbounded acoustic problems and can handle very complex geometries. The idea behind the hybrid method is to take advantage of the fast WBM solution for scatterers of moderate complexity and take advantage of the BEM's capability for handling scatterers of high complexity. For the BEM part, the indirect variational formulation is used which allows modeling of open boundary problems (zero thickness walls). In addition, the WBM makes it possible to easily add heterogeneities (domain inclusions) to the problem. Therefore, the hybrid method does not only aim for better efficiency but also for extending the variety of configurations that can be tackled by both methods with ease. The accuracy and the performance of the method are demonstrated with three examples, both in 2D and 3D. It is shown that when complex and simple scatterers coexist, the hybrid method is more efficient than the WBM, the BEM and the Fast-Multipole BEM while it is able to provide accuracy of similar level.



## 1. Introduction

A challenging configuration for unbounded problems, which has a broad range of applications, is the multiple scattering problem. Acoustics, marine engineering, electromagnetism and seismology are some examples of possible application fields. As it is with many engineering areas today, efficient solution of this problem is of great importance.

The challenge in multiple scattering problems stems from predicting complex fields resulting from interaction of incident and reflected waves. When the geometrical complexity of the scatterers themselves is added to the equation, the resulting problem needs extra care. There are various works in literature towards the definition and the solution of this problem. A general overview can be found in [1]. A considerable effort on multiple scattering problems has been concentrated on analytical solutions [2]. The downside of these approaches is that they are limited to simple shapes such that their applicability is limited as well.

For the solution of complex multiple scatterer configurations, the main trend is towards element based methods. Among the element based methods, the domain discretization methods such as the Finite Element Method and the Finite Difference Method do not inherently solve unbounded problems. For this reason, they need special treatments. In addition, for some

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multiple scatterer configurations a vast area in between the scatterers needs to be discretized before these treatments can be applied. Therefore, the conventional domain discretization methods become impractical for many multiple scattering cases. On the other hand, some recent advancements made it possible for the FEM [3] and the FDM [4] to tackle multiple scatterers more efficiently. As opposed to the conventional way of solving unbounded problems with one global artificial boundary enclosing all scatterers, one artificial boundary per scatterer is applied [5,6]. For the non-reflecting boundary conditions, Dirichlet-to-Neumann (DtN) conditions are utilized. The advantage of these approaches is that the model sizes are considerably reduced. Even so, the matrix properties are changed in a way that the resulting matrix loses its sparsity and the evaluation of the DtN functions are time consuming. Moreover, both of the methods suffer from numerical pollution (dispersion) error by their nature [7,8]. Consequently, for mid and high frequency applications, the accuracy of the methods degrades.

The more commonly used element based prediction technique for unbounded applications is the Boundary Element Method (BEM) [9]. Since the boundary integral equation inherently satisfies the Sommerfeld radiation condition, no extra treatment is needed for unbounded problems or multiple scatterer configurations. The mesh generation is rather easy because only the boundaries have to be discretized. The dispersion error is a lot less pronounced compared to domain discretization methods [10]. There are various BEM formulations in literature that aim at different applications. An indirect variational formulation is chosen for the hybrid method because of its ability to tackle open boundary problems. There are also alternative formulations in literature that can do the same [11,12] and the coupling method in this paper would be applicable to them as well. The disadvantage of the BEM is that, there are singularities in the integral equations so extra care is needed to evaluate them properly. Also, the BEM results in fully populated and complex matrices which can be time consuming to solve for large problems. On the other hand, accelerated BEM formulations such as Fast Multipole BEM (FM-BEM) [13,14] aim to reduce calculation times while using the same fundamental governing equations of the BEM. The idea is to reduce the cost by approximating the effect of far elements, thus avoiding unnecessary evaluations which have minor effects on the results. The method avoids building of the full BEM matrix and uses vector–matrix products and iterative solvers for efficient solutions. The speed gain can be considerably high, however this is only possible for rather big models because the multipole algorithms themselves are time consuming. Therefore, for small models the BEM can still be faster.

While conventional element based methods are still popular and provide a wide range of opportunities, new alternative deterministic techniques or extensions to element based methods are also getting attention. The Method of Fundamental Solutions [15,16], the Equivalent Source Techniques [17], the Full-Field Method [18], the Wave BEM [19] and the Wave Based Method [20] can be given as examples. Among those, the Wave Based Method is the main focus in this work. It adopts a Trefftz approach [21] and makes use of shape functions which are exact solutions of the underlying homogeneous differential equations. This approach, being in contrast with the approximating, simple polynomials used in element based techniques, yields small numerical models with a high convergence rate and has the potential to tackle mid-frequency dynamic problems [22].

So far, the WBM in its pure form has been used for the efficient analysis of problems of moderate geometrical complexity in areas such as the steady-state analysis of acoustic and vibro-acoustic problems (bounded and unbounded) [23–26], porous materials [27] and structural dynamic problems [28]. For more complex problems in the bounded (vibro-)acoustic field, a hybrid form with the FE has been developed [29]. This paper concentrates on 2D and 3D complex unbounded acoustic applications.

For bounded acoustic problems, a sufficient condition for the WBM to converge is the convexity of the considered acoustic domain. For non-convex geometries, the considered problem domain has to be partitioned into convex subdomains. The WBM shows its full power when the partitioning results in a small number of large WB subdomains rather than a large number of small subdomains. As such, the method is more suitable for problems of moderate geometrical complexity.

For 2D unbounded problems, a global circular truncation line and for 3D unbounded problems, a global spherical surface are used to enclose the problem geometry and the area in between is modeled using bounded subdomains. Therefore, the modeling limitations of the bounded problems concerning the complex geometries are inherited by the unbounded problems. This limitation is more pronounced in multiple scatterer problems where the truncation circle/sphere should enclose all the scatterers at once and the area in between requires a complex subdomain division.

To remedy this, the concept of Multi-Level WBM (ML-WBM) was established [30]. The main idea is to divide the problem domain into so called 'levels' such that each level contains one scatterer and forms a sub-problem. The superposition principle is then used to couple the levels through a weighted residual formulation and build the total system of equations. As a result, instead of using one global truncation circle/sphere $\Gamma_t^g$ enclosing all the scatterers, every scatterer is enclosed by a close fitting truncation circle/sphere $\Gamma_{t,i}$ in its corresponding level which saves a great modeling effort for multiple scatterer problems (see Section 4). Nevertheless, the geometrical limitation of the WBM still shows its presence when one of the scatterers has a highly complex shape and requires many subdomains within its isolated level. The efficiency and the accuracy of the method are impaired and it loses its competitiveness.

Besides the presented methods, there are also some examples in literature where different methods are coupled to benefit from the better parts of each. Hampel et al. [31] couple the BEM to a ray tracing procedure to add the effect of refraction at long distances from the source. Tadeu et al. [32] couple the BEM and the Traction BEM to the Method of Fundamental Solutions and demonstrate the hybrid method's capability to tackle thin fluid-filled inclusions. They also converted the solution from the frequency domain to the time domain and present transient analysis results.