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A new scheme of causal viscous hydrodynamics for relativistic heavy-ion collisions: A Riemann solver for quark–gluon plasma

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ABSTRACT

In this article, we present a state-of-the-art algorithm for solving the relativistic viscous hydrodynamics equation with the QCD equation of state. The numerical method is based on the second-order Godunov method and has less numerical dissipation, which is crucial in describing of quark–gluon plasma in high-energy heavy-ion collisions. We apply the algorithm to several numerical test problems such as sound wave propagation, shock tube and blast wave problems. In sound wave propagation, the intrinsic *numerical* viscosity is measured and its explicit expression is shown, which is the second-order of spatial resolution both in the presence and absence of *physical* viscosity. The expression of the numerical viscosity can be used to determine the maximum cell size in order to accurately measure the effect of physical viscosity in the numerical simulation.

1. Introduction

A relativistic fluid approach has been applied to various high-energy phenomena in astrophysics, nuclear, and hadron physics, bringing a lot of interesting and outstanding results. In particular, recent relativistic hydrodynamic analyses revealed a new and interesting feature of quark–gluon plasma (QGP) in high-energy heavy-ion collisions. Since the Relativistic Heavy-Ion Collider (RHIC) at Brookhaven National Laboratory (BNL) started operation in 2000, a number of discoveries have been made, providing insight into quantum chromodynamics (QCD) phase transition and the QGP. One of the most interesting and surprising outcomes at RHIC was the production of the strongly interacting QGP (sQGP), which was confirmed by both theory and experiment. The highlights are: (i) strong elliptic flow, which suggests that collectivity and thermalization are achieved; (ii) strong jet quenching, which confirms that hot and dense matter is created after collisions; (iii) the quark number scaling of elliptic flow, which indicates that the hot quark soup is produced [1,2]. Relativistic hydrodynamic models have made a significant contribution to these achievements. For example, at the time, only hydrodynamic models could explain the strong elliptic flow at RHIC, which was considered to be direct evidence for the production of sQGP at RHIC. Because of the success of the relativistic hydrodynamic model at RHIC, hydrodynamic analysis has become a useful and powerful tool for understanding dynamics of hot and dense matter in high-energy heavy-ion collisions.

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Table 1
Ideal hydrodynamical models. In the table, we use the following abbreviation. lQCD: lattice QCD inspired EoS, SPH: smoothed particle hydrodynamics, PPM: piecewise parabolic method.

Ref.	Dimension	EoS	Numerical scheme
Hama et al. [10]	$3+1$	Bag model	SPH
Hirano et al. [11]	$3+1$	Bag model	PPM
Nonaka and Bass [12]	$3+1$	Bag model	Lagrange
Hirano et al. [13,14]	$3+1$	lQCD	PPM
Petersen et al. [15]	$3+1$	lQCD	SHASTA
Karpenko and Sinyukov [16]	$3+1$	lQCD	HLLE
Holopainen et al. [17]	$2+1$	lQCD	SHASTA
Pang et al. [18]	$3+1$	lQCD	SHASTA

In the early stage of the hydrodynamic studies at RHIC, viscosity effects were not taken into account. However, detailed analyses of experimental data in relativistic heavy-ion collisions gradually revealed limitation of ideal hydrodynamic models. In Ref. [3], for the first time, quantitative analyses of elliptic flow were performed with a relativistic viscous hydrodynamic model. The authors showed that ideal hydrodynamics overestimates elliptic flow as a function of transverse momentum, and that a hydrodynamic calculation with finite viscosity explains the experimental data better. Since then, the main purpose of the phenomenological study for relativistic heavy-ion collisions at RHIC and LHC has been to obtain detailed information of bulk properties of QGP, such as its transport coefficients. Besides, recent high statistical experimental data at RHIC and LHC require more rigorous numerical treatment on the hydrodynamical models. Recently both at RHIC and LHC the higher harmonic anisotropic flow, which is the Fourier coefficient of particle yield as a function of azimuthal angle, has been reported. One of the origins of the higher harmonics is event-by-event fluctuations. To obtain the precise value of transport coefficients with relativistic viscous hydrodynamics, we need to choose an algorithm with small numerical dissipation and treat the inviscid part with care. Usually each algorithm has advantages or disadvantages in terms of coding, computational time, precision and stability. Thus far, unfortunately, only limited attention has been paid to numerical aspects in hydrodynamic models for high-energy heavy-ion collisions.

In this article, we present a state-of-the-art algorithm for solving the relativistic viscous hydrodynamics equation with the QCD equation of state (EoS). Our applications require a numerical scheme that can treat a shock wave appropriately and has less numerical dissipation in order to gain comprehensive understanding of recent high-energy heavy-ion collision physics. These advantages can be achieved by implementing a Riemann solver for the relativistic ideal hydrodynamics. In particular, we propose a new Riemann solver for the QCD EoS at low baryon density, which has not been considered in astrophysical application where baryon density is usually much higher. We derive our Riemann solver by analytically solving the relativistic Riemann problem for low baryon density, within the approximation scheme proposed by [4]. As we will see in Section 5, where we perform several numerical tests, our new algorithm with the Riemann solver has an advantage over other algorithms such as Kurganov–Tadmor (KT) [5], Nessyahu–Tadmor (NT) [6] and SHASTA [7] from the point of view of analyses for current relativistic heavy-ion collisions. By implementing our new Riemann solver for relativistic ideal hydrodynamics in a numerical scheme for causal viscous hydrodynamics recently proposed in Ref. [8], we can also construct a new algorithm for causal viscous hydrodynamics for QGP.

This article is organized as follows. In Section 2, we review current hydrodynamic models for relativistic heavy-ion collisions and introduce the basics of relativistic hydrodynamics. In Section 3, we explain the QCD EoS at high temperature and low baryon density based on the latest lattice QCD calculation. In Section 4, we propose a new Riemann solver for the ideal fluid with the QCD EoS at high temperature and low baryon density. In Section 5, using the numerical scheme, we show results of several numerical tests, such as sound wave propagation, as well as shock tube and blast wave problems. Section 6 is devoted to summary and discussions. In this article, we adopt natural units, with the speed of light in vacuum $c=1$, Boltzmann constant $k_B=1$ and Planck's constant $\hbar=1$.

2. Hydrodynamic models

First we list current hydrodynamic models, which are applied to relativistic heavy-ion collisions [9] in Tables 1 and 2. Here we mention the key aspects of numerical simulations in relativistic hydrodynamic models, which are classified into ideal versions and viscous ones. One of the important ingredients of hydrodynamic models is an EoS, needed for solving the relativistic hydrodynamics equation. Different types of physics related to QCD phase transitions can be input into the EoS.[1] From comparison between hydrodynamic calculations and experimental data of high-energy heavy-ion collisions, the information for the QCD phase diagram is obtained through the EoS used in the hydrodynamic calculation. The Bag model type EoS with the first-order phase transition has been widely used in relativistic hydrodynamic models, because of its simplicity and the lack of conclusive results on EoS of QCD. In recent hydrodynamical calculations, lattice-inspired EoS has

[1] For the further application to relativistic heavy-ion collisions, not only EoS but also other aspects should be discussed; initial conditions and final conditions (freeze-out processes and final state interactions) of the hydrodynamic simulation. Since modeling of these aspects is beyond the scope of this paper, they are not addressed in Tables 1 and 2.