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An efficient correction procedure via reconstruction for simulation of viscous flow on moving and deforming domains

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ABSTRACT

In this paper, we report the development of a new parallel solver using the Correction Procedure via Reconstruction (CPR) for viscous flows on moving and deforming grids. By employing an accurate treatment of flux derivatives for moving and deforming unstructured grids consisting of all quadrilateral cells, it is found that the Geometric Conservation Law is not explicitly required, the free-stream preservation is automatically satisfied. *The CPR code is verified using a benchmark case for a moving inviscid vortex on moving and deforming grids. The optimal orders of accuracy are obtained. It is subsequently employed to study viscous flows on moving and deforming grids. The CPR method is faster than and nearly as accurate as the SD method for solving viscous flow problems with moving boundaries*.

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1. Introduction

In 2007, Huynh [5] introduced a new approach to high-order accuracy by solving the equations in differential form. The approach, originally called flux reconstruction (FR), results in numerous schemes with favorable properties including an earlier developed spectral difference (SD) method [9,14] for quadrilateral elements. In 2009, Wang and Gao [20] extended the FR idea to 2D triangular and mixed meshes with the lifting collocation penalty (LCP) framework. The involved authors later combined the names of "FR" and "LCP" to call them CPR (Correction Procedure via Reconstruction). The CPR formulation is believed to be among the most efficient discontinuous methods in terms of the number of operations in Wang et al. [21]. *Correction functions are used to correct the discontinuous flux function within an element in order to ensure flux continuity across element interfaces. The g_2 scheme is the most efficient method in the family of CPR methods. The correction function of the g_2 scheme proposed by Huynh [6] and further developed here for moving and deforming domains is expressed in terms of a combination of Radau polynomials, where the zeros of the derivative of the correction function coincide with the Legendre–Lobatto points. Recently, Vincent et al. [19] reported a stability proof using a flux reconstruction formulation with energy estimates for a one parameter family of schemes where the family was expressed in terms of the Legendre polynomials instead of the Radau polynomials, proving stability for all orders of accuracy on unstructured grids*. Recently, Liang et al. [12] reported that the 4th-order CPR method could be over 40% faster than the SD method for viscous flow on 2D stationary grids with all quadrilateral cells.

While the SD or CPR method is quite established for simulations on stationary grids with complex geometries, its potential for simulating unsteady flow on moving and deforming domains could be very helpful for modeling flows associated with flapping wing flights, micro-air vehicles, oscillating wing energy harvesters, etc. To the best knowledge of the authors, the CPR method has not yet been applied for computations on moving and deforming grids. *The SD method is a staggered*

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method requiring two grids for distributing flux and solution points at different locations. We consider a particular CPR method to co-locate the flux and solution points one-on-one within the standard element. The resulting discrete Geometric Conservation Law is satisfied automatically. In this paper, we extend the g_2 scheme originally proposed in Huynh [5,6] to solve both inviscid and viscous flows on moving and deforming grids consisting of all quadrilateral cells. By employing an accurate treatment of flux derivatives, we find that an enforcement of the Geometric Conservation Law by adding source terms [16] for free-stream preservation is no longer necessary.

The paper is arranged as the following. Section 2 presents the mathematical formulation of equations in both physical and computational domains. Section 3 gives the numerical formulation of the CPR method on moving and deforming grids. In Section 4, we test the accuracy of CPR method by solving the inviscid Euler vortex propagation problem on stationary and deforming grids. In Section 5, we validate our methods and solver by studying viscous flow over an oscillating cylinder, we also test the efficiency of CPR method on moving and deforming grids as well as the scalability of the parallelization. After that, in Section 6 and Section 7, we further validate the solver through the study of viscous subsonic flows around pitching and plunging airfoils. Section 8 concludes this paper.

2. Mathematical formulation

We consider compressible Euler as well as Navier–Stokes equations in 2D. The conservative form is given by

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = 0, \tag{1}$$

where $\mathbf{Q}$ is the vector of conserved variables; $\mathbf{F}$ and $\mathbf{G}$ are flux vectors in two Cartesian directions which can include both inviscid and viscous flux terms.

In both CPR and SD methods, we employ an iso-parametric mapping to transform the fully-conservative equations from the physical domain onto a computational domain which allows universal reconstruction via polynomials. The computational domain is represented by a standard square element $0 \leqslant \xi \leqslant 1$, $0 \leqslant \eta \leqslant 1$, and a pseudo time τ. The Jacobian matrix for the mapping is,

$$\mathcal{J} = \frac{\partial(x, y, t)}{\partial(\xi, \eta, \tau)} = \begin{bmatrix} x_\xi & x_\eta & x_\tau \\ y_\xi & y_\eta & y_\tau \\ t_\xi & t_\eta & t_\tau \end{bmatrix}, \tag{2}$$

where $t_\xi = 0$, $t_\eta = 0$, and $t_\tau = 1$.

The inverse of matrix $\mathcal{J}$ is

$$\mathcal{J}^{-1} = \frac{\partial(\xi, \eta, \tau)}{\partial(x, y, t)} = \begin{bmatrix} \xi_x & \xi_y & \xi_t \\ \eta_x & \eta_y & \eta_t \\ \tau_x & \tau_y & \tau_t \end{bmatrix} = \frac{1}{|\mathcal{J}|}\mathcal{S}, \tag{3}$$

where $\tau_x = 0$, $\tau_y = 0$, and $\tau_t = 1$. And $\mathcal{S}$ is the transpose of the cofactor matrix of $\mathcal{J}$ which can be written as

$$\mathcal{S} = \begin{bmatrix} y_\eta & -x_\eta & A \\ -y_\xi & x_\xi & B \\ 0 & 0 & |\mathcal{J}| \end{bmatrix}, \tag{4}$$

with $A = x_\eta y_\tau - x_\tau y_\eta$, $B = x_\tau y_\xi - x_\xi y_\tau$ and $|\mathcal{J}| = x_\xi y_\eta - x_\eta y_\xi$.

The Geometric Conservation Law (GCL) can be expressed as

$$\begin{cases} \dfrac{\partial}{\partial \xi}\big(|\mathcal{J}|\xi_x\big) + \dfrac{\partial}{\partial \eta}\big(|\mathcal{J}|\eta_x\big) = 0, \\ \dfrac{\partial}{\partial \xi}\big(|\mathcal{J}|\xi_y\big) + \dfrac{\partial}{\partial \eta}\big(|\mathcal{J}|\eta_y\big) = 0, \\ \dfrac{\partial |\mathcal{J}|}{\partial \tau} + \dfrac{\partial}{\partial \xi}\big(|\mathcal{J}|\xi_\tau\big) + \dfrac{\partial}{\partial \eta}\big(|\mathcal{J}|\eta_\tau\big) = 0. \end{cases} \tag{5}$$

The first two equations are automatically satisfied since the iso-parametric mapping is analytic and the metrics can be calculated exactly. We only have to consider the last equation which is time dependent.

In the following, we first give the classical form of Eq. (1) in the computational domain as was reported in Yu et al. [22] and Liang et al. [11]. Then we present the equation in a new form, namely the Liang–Miyaji form, which has been used in this paper.