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A comparison of SPH schemes for the compressible Euler equations

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ABSTRACT

We review the current state-of-the-art Smoothed Particle Hydrodynamics (SPH) schemes for the compressible Euler equations. We identify three prototypical schemes and apply them to a suite of test problems in one and two dimensions. The schemes are in order, standard SPH with an adaptive density kernel estimation (ADKE) technique introduced Sigalotti et al. (2008) [44], the variational SPH formulation of Price (2012) [33] (referred herein as the MPM scheme) and the Godunov type SPH (GSPH) scheme of Inutsuka (2002) [12]. The tests investigate the accuracy of the inviscid discretizations, shock capturing ability and the particle settling behavior. The schemes are found to produce nearly identical results for the 1D shock tube problems with the MPM and GSPH schemes being the most robust. The ADKE scheme requires parameter values which must be tuned to the problem at hand. We propose an addition of an artificial heating term to the GSPH scheme to eliminate unphysical spikes in the thermal energy at the contact discontinuity. The resulting modification is simple and can be readily incorporated in existing codes. In two dimensions, the differences between the schemes is more evident with the quality of results determined by the particle distribution. In particular, the ADKE scheme shows signs of particle clumping and irregular motion for the 2D strong shock and Sedov point explosion tests. The noise in particle data is linked with the particle distribution which remains regular for the Hamiltonian formulations (MPM and GSPH) and becomes irregular for the ADKE scheme. In the interest of reproducibility, we make available our implementation of the algorithms and test problems discussed in this work.

1. Introduction

Smooth Particle Hydrodynamics (SPH) is a mesh-free particle method for the solution of the continuum equations of physics. Although it was introduced by Lucy [20] and subsequently developed by Gingold and Monaghan [11] for problems in astrophysics, it has evolved into a versatile tool that is capable of solving a wide class of problems ranging from gas dynamics to solid mechanics. A number of researchers have provided excellent reviews of the development of SPH and it's myriad applications [24,26,36,35]. In this work, we are concerned with the application of SPH to the compressible Euler equations. To this end, we evaluate three popular SPH schemes for their robustness and accuracy when applied to a suite of shock-tube like test problems in one and two dimensions. We begin with a brief overview of the development of SPH applied to the Euler equations.

Gingold and Monaghan [27] were the first to successfully apply SPH to the shock tube-problem. The Euler equations in Lagrangian form, governing the material rate of change of primitive variables were discretized using SPH approximation

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techniques. An artificial viscosity term, chosen to model a bulk viscosity in the continuum limit was used to handle discontinuities in the flow. The artificial viscosity method was successful in solving the relatively simple shock-tube problem, producing accurate post-shock values and negligible oscillations. A salient feature of SPH simulations of shock tube phenomena, a "blip" in the pressure profile at the contact discontinuity was observed [27]. This anomaly in an otherwise accurate solution was attributed to the way in which the energy equation is handled by SPH. Presumably, SPH struggles with the initial discontinuous thermal energy, resulting in this kink in pressure, which neither grows nor attenuates without dissipation. Experiments with more challenging test cases, particularly, high Mach number collisions revealed that the SPH particles may stream past each other. Lattanzio et al. [17] solved this problem by adding a von Neumann–Richtmyer type artificial viscosity term [51], as a viscous pressure to the momentum equation and thereby preventing penetration. The resulting viscosity term is what is now referred to as the standard form of artificial viscosity used in SPH [23,24]. These early implementations of SPH used a constant smoothing length and we will refer to this formulation (in particular, Monaghan [24]) hereafter as the "standard" SPH. The artificial viscosity introduced by Monaghan had a number of desirable features in the context of a particle method like SPH. The inter-particle viscous contribution is symmetric in particle indices and acts along the line joining them. These requirements are pivotal to preserving the global conservation properties of the discrete SPH system. The viscosity vanishes for rigid body rotation and is Galilean invariant, a property that mimics the governing equations. The drawbacks of this viscosity are that it does not vanish for shearing flows and produces too much diffusion away from shocks. Balsara [8] proposed a switch to limit the viscosity in regions of high shear which is commonly employed for shearing flows encountered in astrophysics.

Monaghan [25] introduced new viscous terms by constructing them in a manner analogous to dissipative terms arising in Eulerian FVM schemes. These schemes define waves (eigenvectors) and wave speeds (eigenvalues) from the Jacobian matrix and changes across these waves are used to construct the dissipative terms. In SPH, the line joining two particles are used as the eigenvector and jumps in relevant physical quantities along the line joining two particles define the wave jump. The analogy is completed by defining a "signal" velocity, which determines the speed (eigenvalue) with which information is exchanged by two interacting particles. Sufficient care is taken to preserve symmetry and to ensure that the viscous contribution to the thermal energy is positive definite. While the viscosity was shown to work well for a suite of one-dimensional problems, the arguments leading to their development were heuristic as pointed out by Monaghan himself. The problem of excessive dissipation away from shocks was addressed by Morris and Monaghan [28], who introduced the concept of solution-dependent viscosity coefficients. The idea is to increase the coefficient of viscosity in regions of compression while reducing them elsewhere. A reliable shock sensor is needed which is usually derived from the velocity divergence [28,33,18]. The scheme was further developed by Price and Monaghan [34] to solve MHD problems. An adaptive smoothing length, consistent with the density estimate is chosen which aims to keep the mass in a smoothing sphere approximately constant. An action principle is used to derive the unique equations of motion and the "grad-h" (∇h) terms [29], arising from the non-linear density-smoothing length relation are included for better accuracy in the solution. This scheme is described in considerable detail by Price [33] and an implementation (NDSPMHD) is available for validation and use. In this work, we refer to this scheme as the Monaghan, Price and Morris (MPM) scheme.

Hernquist [16] demonstrated that a certain duality exists in SPH with respect to energy conservation when using variable smoothing lengths. In particular, evolving the thermal energy equation leads to good energy conservation but entropy is not conserved. Likewise, integrating the entropy equation results in poor energy conservation. In light of this, Springel and Hernquist [50] used a variational technique to develop an SPH scheme that correctly accounts for the "grad-h" terms when the smoothing lengths are updated using an iterative scheme aimed at keeping the number of neighbors approximately constant [25]. To control the rate of entropy generation across shocks, an entropy variable is evolved instead of the thermal energy. In a recent review article, Springel [49] discusses this formulation and it's advantages for problems in astrophysics. An implementation (GADGET [48]) is available for general use. Our experiments with this formulation indicate that the NDSPMHD and GADGET codes produce nearly identical results for the Euler equations, with the main difference being a large pressure "blip" at the contact discontinuity for GADGET. This is to be expected since GADGET does not incorporate any explicit thermal conduction which is known to smoothen this anomaly in the SPH solution. Moreover, it is known that in the variational formulation, evolving the entropy, thermal energy or the specific energy are equivalent to within integration errors [33]. The two codes therefore, differ only in the way in which the energy equation is evolved. Recently, Abel [45] has suggested a discretization of the momentum equation that applies forces to particles only when there is a net pressure force acting upon them. Since the new momentum equation depends on the relative pressure between two particles, the formulation is dubbed rpSPH. The change is easily incorporated into existing SPH codes (in particular, GADGET). While leading to better estimate for the gradient of pressure, the formulation breaks the pair-wise symmetry of particle interactions which are pivotal for conservation arguments in SPH. Price [33] has demonstrated that such a non-conservative discretization leads to an unfavorable particle clumping behavior for SPH simulations. We do not include rpSPH in our comparison as there is no evidence to suggest that it has an advantage for highly compressible flows [45].

An extension of the standard SPH albeit with a new recipe to update the smoothing length was proposed by Sigalotti et al. [43,44]. Instead of solving the non-linear density-smoothing length equation, an adaptive density kernel estimation (ADKE) procedure is used to update the smoothing lengths. The ADKE technique uses the ratio of the local particle density to a global average to obtain smoothly varying "bandwidth" factors that are used to update the smoothing length. The method is in principle identical to the algorithm proposed by Steinmetz and Müller [21], with the difference being the way the global average is computed. The resulting method works well and is easy to implement, however, the method