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Beyond first-order finite element schemes in micromagnetics

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ABSTRACT

Magnetization dynamics in ferromagnetic materials is ruled by the Landau–Lifshitz–Gilbert equation (LLG). Reliable schemes must conserve the magnetization norm, which is a nonconvex constraint, and be energy-decreasing unless there is pumping. Some of the authors previously devised a convergent finite element scheme that, by choice of an appropriate test space – the tangent plane to the magnetization – reduces to a linear problem at each time step. The scheme was however first-order in time. We claim it is not an intrinsic limitation, and the same approach can lead to efficient micromagnetic simulation. We show how the scheme order can be increased, and the nonlocal (magnetostatic) interactions be tackled in logarithmic time, by the fast multipole method or the non-uniform fast Fourier transform. Our implementation is called feeLLGood. A test-case of the National Institute of Standards and Technology is presented, then another one relevant to spin-transfer effects (the spin-torque oscillator).

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1. Introduction

The Landau–Lifshitz–Gilbert equation (LLG) describes magnetization dynamics in submicronic ferromagnetic materials. According to micromagnetic theory, the magnetization is a continuous vector field $M(x,t) : \Omega \times [0,T] \to \mathbb{R}^3$ that locally moves along a damped precession around an effective magnetic field H_{eff}. The equation writes:

$$\partial_t M - \frac{\alpha}{M_s} M \times \partial_t M = -\gamma_0 M \times H_{\text{eff}} \tag{1}$$

where γ_0 is called the gyromagnetic ratio and $M_s = |M|$ the saturation magnetization, and $\alpha > 0$ is the damping parameter. It follows from the equation that $|M|$ is conserved in time. Several energy terms contribute to the effective field, traditionally exchange, magnetostatic, magnetocrystalline and externally applied fields [1]. Exchange is of quantum origin and responsible for the ferromagnetic order; it is a Laplacian field. Other fields are operators of order at most one, and we refer to them collectively as H_r. The magnetostatic field in particular is nonlocal. A spin-torque field has recently been added in order to model spin-polarized current injection [2]. Denoting $m = M/M_s$ and $\partial_t m$ its derivative with respect to the dilated time $\gamma_0 t$, the motion of m in the unit sphere is thus given by

$$\partial_t m - \alpha m \times \partial_t m = -m \times H_{\text{eff}}. \tag{2}$$

In [3] and [4] the equation was defined in a weak sense, i.e. for $m \in H^1(\Omega, S^2)$, testing it against smooth functions $w \in H^1(\Omega, R^3)$ by $(m,w) = \int_\Omega m \cdot w$. Finite element approximations are globally continuous piecewise polynomials

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satisfying some natural conditions. As usual, test functions that vanish at all but one node express local averages [5]. And of course, the finite element method (FEM) can handle complex geometries. Micromagnetic codes developed in the physics community usually solve (2) by the finite difference method [6]. Some, like magpar [7] and Nmag [8], use the FEM to reconstruct H_{eff} at mesh nodes, reducing (2) to an ODE. No finite element approximation of m is built, and H_{eff} is artificially smoothed at the interface of two materials where it may be discontinuous.

A finite element approximation with nice properties was devised by Alouges [9] for exchange only, then generalized to all energy terms by some of the authors [10]. Noting that m and $\partial_t m$ are orthogonal, the latter was sought in the tangent plane to the magnetization m^n at the n-th time step. The resulting problem is linear, as nonlinearity is hidden in the normalization done at each step. However, the scheme is of order 1. Here we aim to show that this is not an intrinsic limitation, and that the same approach can be used for efficient micromagnetic simulation. The paper is organized as follows:

Section 2 recalls the finite element scheme and shows how to get beyond order 1 while still solving a linear problem. Section 3 addresses the issue of the nonlocal magnetostatic field, where we prefer fast summation methods over the boundary element method for efficiency on flat geometries. Our implementation, called feeLLGood (finite element LLG object oriented development) is tested in Section 4; first (4.1) on the Standard Problem #4 of the NIST, then (4.2) on the spin-torque oscillator previously studied by the finite difference code ST_GL-FFT [6].

2. The finite element scheme

Let us recall the scheme found in [9] and discuss stability, keeping the term of highest order in H_{eff} – the exchange. It was proved in [10] that the main results hold when other energy terms are added (the central argument is that H_r is bounded in L^p, $1 < p < +\infty$). Then taking the cross product of (2) by m gives the sphere-valued harmonic map flow [11] with precession:

$$\alpha\,\partial_t m + m \times \partial_t m = \Delta m + |\nabla m|^2 m. \tag{3}$$

Note that we don't write the physical constant before the Laplacian. The related Dirichlet energy is $\frac{1}{2}\|\nabla m\|^2$. The last term means $\partial_t m$ is tangent to m hence to the unit sphere, and may be called the geometric term.

2.1. The first-order scheme

Based on the last remark, Alouges and Jaisson [12], introduced a suitable finite element space, the tangent plane

$$T^n = \left\{ v = \sum_i v_i \phi_i,\ v_i \cdot m_i = 0 \right\} \tag{4}$$

of vector fields tangent to m^n at mesh nodes, where the ϕ_i are the basis linear functions adapted to the triangulation. The point is that when tested against $w \in T^n$, the geometric term vanishes. For example, calling the time step k, the explicit scheme writes:

1. find $v \in T^n$ such that for all $w \in T^n$,

$$\alpha(v, w) + \left(m^n \times v, w\right) = -\left(\nabla m^n, \nabla w\right); \tag{5}$$

2. normalize $m^n + kv$ to get m^{n+1}:

$$m^{n+1} := \frac{m^n + kv}{|m^n + kv|}. \tag{6}$$

Owing to step 2, the norm constraint is satisfied at all times. As usual, the stability of the explicit scheme is subject to a Courant condition (see below). It is improved by applying a θ-scheme for the Laplacian [9], i.e. replacing m^n with $m^n + \theta k v$, $0 \leqslant \theta \leqslant 1$, there, thus adding a stiffness term to (5):

$$\alpha(v, w) + \left(m^n \times v, w\right) + \theta k(\nabla v, \nabla w) = -\left(\nabla m^n, \nabla w\right). \tag{7}$$

2.2. Stability analysis

Taking $w = v$ in (7) yields the power dissipation

$$\left(\nabla m^n, \nabla v\right) = -\alpha\|v\|^2 - \theta k\|\nabla v\|^2 \tag{8}$$

so that

$$\frac{1}{2}\left\|\nabla\left(m^n + kv\right)\right\|^2 = \frac{1}{2}\left\|\nabla m^n\right\|^2 - \alpha k\|v\|^2 - \left(\theta - \frac{1}{2}\right)k^2\|\nabla v\|^2. \tag{9}$$

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