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Entropy/energy stable schemes for evolutionary dispersal models

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ABSTRACT

In this paper we propose some entropy/energy stable finite difference schemes for the reaction–diffusion–advection equation arising in the evolution of biased dispersal of population dynamics. The peculiar feature of these active dispersal models is that the transient solution converges to the stable steady state when time goes to infinity. For the numerical method to capture the long-time pattern of persistence or extinction, we use the relative entropy when the resource potential is logarithmic, and explore the usual energy for other resource potentials. The present schemes are shown to satisfy three important properties of the continuous model for the population density: (i) positivity preserving, (ii) equilibrium preserving, and (iii) entropy or energy satisfying. These ensure that our schemes provide a satisfying long-time behavior, thus revealing the desired dispersal pattern. Moreover, we present several numerical results which confirm the second-order accuracy for various resource potentials and underline the efficiency to preserve the large time asymptotic.

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1. Introduction

This work is concerned with the numerical approximation of a class of reaction–diffusion–advection equations arising in the evolution of biased dispersal of population dynamics, with emphasis on exploring the entropy/energy structure of the dispersal model so that the resulting methods provide a satisfying long-time behavior.

1.1. Mathematical formulations

Reaction–diffusion equations have been widely used to model the biological problems [35,36]. One of the well-known examples is the logistic reaction–diffusion model for the population growth with random dispersal,

$$\partial_t u = \Delta u + \lambda u(m-u) \quad \text{in } \Omega \times (0,\infty), \tag{1.1}$$

where the population inhabiting a bounded domain $\Omega \subset \mathbb{R}^d$ has density $u(x,t)$ at location x and time t, and local growth rate m. The parameter $\lambda > 0$ is the reverse of the dispersal rate.

If the environment is heterogeneous, i.e., $m(x)$ is not a constant, then the population may have a tendency to move toward resources in addition to the random movement. The model may be upgraded to the following form

$$\partial_t u = \nabla \cdot (\nabla u + u\nabla P) + \lambda u(m-u) \quad \text{in } \Omega \times (0,\infty), \tag{1.2}$$

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where $P = P(m)$, which we call resource potential, reflects the movement tendency of the population. The time evolution is subjected to both the initial density $u(x,0) = u_0(x)$ and the zero flux boundary condition

$$(\nabla u + u\nabla P)\cdot \nu = 0 \quad \text{on } \partial\Omega \times (0,\infty), \tag{1.3}$$

where ν is the outward normal vector on the boundary $\partial\Omega$ which is assumed to be smooth. Several dispersal strategies have been studied in literature, for example $P = -\alpha m$ in [2,5,9,13,15], and the obtained results may apply to a more general reaction than

$$F(u,m) = u(m-u),$$

as long as it satisfies $F(0,m) = 0$ and $F(+\infty,m) < 0$. With the logistical reaction term or its variation, the biologically relevant solutions to (1.2) are nonnegative and ultimately bounded. It follows from the regularity theory of parabolic partial differential equations that in the state space bounded orbits are pre-compact, hence the semi-flow will have a compact attractor, and the ω-limit set of any initial state $u_0(x)$ will be a compact invariant set. In other words, as time evolves the solution of (1.2) is expected to approach some stable patterns, independent of the choice of initial density; see, e.g., [7]. The main result reviewed in Section 2 may be stated as follows.

Theorem 1.1. *Suppose that $m \in C^2(\bar{\Omega})$ is positive somewhere in Ω and P is smooth in m. There exists a unique $\lambda^* > 0$ and a positive equilibrium solution to (1.2) if and only if*

$$\int_\Omega me^{-P}\,dx < 0.$$

Moreover, $\lambda^* = 0$ if and only if $\int_\Omega me^{-P}\,dx \geqslant 0$.

(1) *If $0 < \lambda \leqslant \lambda^*$, all nonnegative solutions of (1.2) decay toward zero as $t \to \infty$.*
(2) *If $\lambda > \lambda^*$, the positive equilibrium is globally attractive among nonzero nonnegative solutions.*

Of special interest is the ideal free dispersal strategy determined by P such that at an equilibrium $u_{eq}(x)$ [8,10,11], both

$$\nabla\cdot(\nabla u_{eq} + u_{eq}\nabla P) = 0 \quad \text{and} \quad m - u_{eq} = 0 \quad \text{in } \Omega.$$

This will hold if $P = -\ln m$ (unique up to some constant). In such a setting, the species u can perfectly match the environmental resource. The role of ideal free distributions has been well recognized in literature. Moreover, the quantity

$$E[u] = \int_\Omega \left[u\log\frac{u}{m} + m - u\right]dx \tag{1.4}$$

is nonincreasing in time. This corresponds to some physical entropy relative to the equilibrium state $u_{eq}(x) = m(x)$, called relative entropy. For general resource potential P, the quantity

$$V[u] = \int_\Omega \left[\frac{1}{2}e^{-P}\left|\nabla\left(ue^P\right)\right|^2 - \lambda e^P \int_0^u F(\xi,m)\,d\xi\right]dx \tag{1.5}$$

is nonincreasing in time. Moreover, Eq. (1.2) can be rewritten as a gradient flow,

$$\partial_t u = -e^{-P}\frac{\delta V}{\delta u}, \tag{1.6}$$

where $\frac{\delta V}{\delta u}$ denotes the standard variational derivative of V with respect to u, and the functional V often corresponds to some free energy of the underlying physics, and thus is called the energy.

In the context of two competing species, the evolutionary dynamics of conditional dispersal becomes much more complex, see [13,22]. A typical model may be described as

$$\begin{cases} u_t = \alpha\nabla\cdot(\nabla u + u\nabla P) + u(m-u-v) & \text{in } \Omega, \\ v_t = \beta\nabla\cdot(\nabla v + v\nabla Q) + v(m-u-v) & \text{in } \Omega, \\ \partial_\nu u + u\partial_\nu P = \partial_\nu v + v\partial_\nu Q = 0 & \text{on } \partial\Omega, \\ u(x,0) = u_0(x), \quad v(x,0) = v_0(x) & \text{in } \Omega. \end{cases} \tag{1.7}$$

The two nonnegative constants α and β represent the dispersal rates of two species, respectively. In this paper we shall focus on the case when $\alpha = \beta$, and we can envision that system (1.7) models two competing species that are identical except their different dispersal strategies P and Q. An interesting question is whether there is any strategy in system (1.7)

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