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A hybrid boundary element-finite element approach to modeling plane wave 3D electromagnetic induction responses in the Earth

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ABSTRACT

A novel hybrid boundary element-finite element scheme which is accelerated by an adaptive multi-level fast multipole algorithm is presented to simulate 3D plane wave electromagnetic induction responses in the Earth. The remarkable advantages of this novel scheme are the complete removal of the volume discretization of the air space and the capability of simulating large-scale complicated geo-electromagnetic induction problems. To achieve this goal, first the Galerkin edge-based finite-element method (FEM) using unstructured meshes is adopted to solve the electric field differential equation in the heterogeneous Earth, where arbitrary distributions of conductivity, magnetic permeability and dielectric permittivity are allowed for. Second, the point collocation boundary-element method (BEM) is used to solve a surface integral formula in terms of the reduced electrical vector potential on the arbitrarily shaped air–Earth interface. Third, to avoid explicit storage of the system matrix arising from large-scale problems and to reduce the horrendous time complexity of the product of the system matrix with an initial vector of unknowns, the adaptive multilevel fast multipole method is applied. This leads to a matrix-free form suitable for the application of iterative solvers. Furthermore, a highly sparse problem-dependent preconditioner is developed to significantly reduce the number of iterations used by the iterative solvers.

The efficacy of the presented hybrid scheme is verified on two synthetic examples against different numerical techniques such as goal-oriented adaptive finite-element methods. Numerical experiments show that at low frequencies, where the quasi-static approximation is applicable, standard FEM methods prove to be superior to our hybrid BEM–FEM solutions in terms of computational time, because the FEM method requires only a coarse discretization of the air domain and offers an advantageous sparsity of the system matrix. At radio-magnetotelluric frequencies of a few hundred kHz, the hybrid BEM–FEM scheme outperforms the FEM method, because it avoids explicit storage of the system matrices as well as dense volume discretization of the air domain required by FEM methods at high frequencies. In summary, to the best of our knowledge, this study is the first attempt at completely removing the air space for large scale complicated electromagnetic induction modeling in the Earth.

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1. Introduction

Plane wave electromagnetic induction methods have become increasingly popular to study the lithospheric structure of the Earth [1] or to investigate the shallow subsurface in engineering and environmental applications [2]. To accurately invert field data for complicated distributions of the electrical parameters of the Earth, there is the need for accurate and efficient algorithms to simulate plane wave electromagnetic induction responses. Currently, there are three numerical techniques which can simulate electromagnetic induction within a complicated 3D heterogeneous Earth, namely finite-difference methods (FDM) [3,4], finite-element methods (FEM) [5–8] and volume integral methods [9,10].

Unfortunately, finite-difference and finite-element methods need to include the volume discretization of the air space in simulations [11]. There are two benefits from removing the volume discretization of the air space from the computational domain. Firstly, the computational domain can be reduced to be predominantly the heterogeneous Earth, which would dramatically decrease the number of unknowns. Secondly, the potential numerical errors arising from the volume discretization of the air space can be eliminated. At high frequencies, the second benefit becomes more critical because here displacement currents play an important role [12]. The propagation characteristics of the electromagnetic fields become so critical that rather dense grids in the air space are required to guarantee acceptable accuracy. The reason is that the wavelengths of the electromagnetic fields in the air space are rather small. To adequately approximate the significant variations of the waves in the air space, careful mesh design in the air space is required. Therefore, the removal of the volume discretization of the air space will avoid serious numerical errors due to improper mesh discretization strategies.

In our previous work [13], a boundary element approach was successfully developed for handling arbitrary surface topography by utilizing surface discretization of the air–Earth interface. However, this BEM cannot deal with the case of arbitrary distributions of the electrical parameters in the subsurface. Therefore, we introduced a finite-element modeling code [14] that deals with arbitrarily complicated models. Combining the benefits of the above two methods, we present here a hybrid BEM–FEM scheme. This hybrid scheme substitutes the volume discretization in the air space by the surface discretization of the air–Earth interface having arbitrary shape and allows for complicated distributions of electrical parameters in the Earth through volume discretization of the subsurface with a FEM.

Unlike other hybrid schemes which were mainly developed for the quasi-static approximation [15,16] or purely high frequency wave propagation problems [17], our hybrid scheme aims to work for a broadband frequency range such as magnetotelluric [18] and radio-magnetotelluric problems [19]. We apply the edge-based finite-element method to solve the electric field equation in the heterogeneous Earth which is discretized by unstructured meshes. Based on the homogeneous electrical properties of the air space, we introduce a surface integral formula which treats the reduced electrical vector potential as its primary variable on the air–Earth interface. The simple but efficient point collocation boundary-element method is used to solve this surface integral formula. These finite-element and surface integral systems are strongly coupled by continuity conditions of the tangential components of both the electric and magnetic fields on the air–Earth interface. To extend its capability to deal with large-scale practical cases [17], an accelerated fast multipole method (FMM) [20] which was specially designed for nonuniform unstructured grids [13] is adopted. Based on the methodology of panel clustering [21], this FMM algorithm can dramatically reduce both the memory cost and computation time on unstructured grids. The core role of this acceleration algorithm is to efficiently compute the product of the system matrix with an initial vector of unknowns, which can be naturally embedded in an iterative solver and leads to a matrix-free scheme that avoids explicit storage of the system matrix. In addition, to accelerate the convergence rate of iterative solvers, a highly sparse and efficient preconditioner is developed.

In the following, we briefly present the theory and two numerical examples to verify and demonstrate our hybrid scheme.

2. Methods

2.1. Hybrid scheme

Three-dimensional (3D) geo-electromagnetic induction modeling requires specification of the geometrical model configuration as shown in Fig. 1. The domain is composed of the air space Ω_0 and the inhomogeneous Earth domain Ω_1 which are connected by the air–Earth interface Γ. A plane wave ($\mathbf{E}_{inc}$, $\mathbf{H}_{inc}$) is incident on the outer surface enclosing the domains Ω_0 and Ω_1, $\partial(\Omega_0 + \Omega_1) = \Gamma_0 + \Gamma_1$. The electrical parameters for Ω_0 and Ω_1 are σ_0, ε_0, μ_0 and σ_1, ε_1, μ_1, respectively, with $\sigma_1 \geqslant \sigma_0$. In a heterogeneous Earth, the parameters are functions of position. We take the positive z-direction as downwards. Our purpose is to compute the distribution of electromagnetic fields in Ω_0 and Ω_1, in terms of the known electromagnetic boundary conditions on surfaces Γ_0 and Γ_1.

The Maxwell equations in the frequency domain (with a time varying factor $\exp^{-i\omega t}$) in the heterogeneous Earth Ω_1 can be written as [22]

$$\nabla \times \mathbf{E}_1 = -\xi_1 \mathbf{H}_1, \tag{1}$$

$$\nabla \times \mathbf{H}_1 = \chi_1 \mathbf{E}_1, \tag{2}$$

$$\nabla \cdot \chi_1 \mathbf{E}_1 = 0, \tag{3}$$

$$\nabla \cdot \xi_1 \mathbf{H}_1 = 0, \tag{4}$$