Journal of Computational Physics 258 (2014) 880–914

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Generalized formulations for the Rhie–Chow interpolation

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A R T I C L E   I N F O

Article history:
Received 22 July 2013
Received in revised form 30 October 2013
Accepted 5 November 2013
Available online 14 November 2013

Keywords:
Pressure-based solver
Staggered-grid discretization
Co-located-grid discretization
SIMPLE schemes
Fourth-order pressure damping
Momentum interpolation
Rhie–Chow interpolation

A B S T R A C T

In this paper, generalized formulations for the Rhie–Chow interpolation for co-located-grid discretizations are derived. These generalized formulations eliminate the major known defects in the standard Rhie–Chow interpolation, including the following: dependence of the converged solution on the value of the under-relaxation factor, saw-tooth pressure oscillations in transient problems with small time steps, and incorrect or non-converged solutions for problems with discontinuities. The generalized formulations are also shown to be applicable to a wider range of flow conditions than the standard Rhie–Chow interpolation. The derivation of the Rhie–Chow interpolation is first recalled and its numerical errors are analyzed. Then, the generalized formulations are presented and explained, and the way in which they eliminate or counter some of the known defects of the standard Rhie–Chow interpolation are outlined. The generalized formulations are then verified and validated with numerical experiments, including experiments with flow in porous media and in a packed bed. It is then concluded that the generalized formulations presented in this work represent an advance over the standard Rhie–Chow interpolation with a negligible increase in computational cost.

1. Introduction

Finite-difference and finite-volume methods for solving the Navier–Stokes equations in the framework of the pressure-based approach [1–4] most commonly use either a staggered-grid or a co-located-grid discretization. Staggered-grid discretizations generally result in greater stability and robustness, but for unstructured and non-orthogonal grids, they are significantly more difficult to implement, especially with respect to the treatment of the boundary conditions, and they require more memory and greater calculation counts. For these reasons, co-located-grid discretizations have been extensively investigated and studied [4–6], and have gained overwhelming popularity over staggered-grid discretizations, especially in general-purpose and commercial flow solvers.

The most significant weakness of co-located-grid discretizations is that they inherently admit checker-board pressure solutions, which are caused by the central-difference discretization of the pressure. About 30 years ago, Rhie and Chow [6] proposed a technique for momentum-based interpolation of the mass fluxes on cell faces. This technique imitates a staggered-grid discretization by forcing the discrete mass conservation equation to be expressed in terms of the discrete mass fluxes across cell faces. For the most part, this technique eliminated the pressure–velocity decoupling and the pressure checker-boarding problem in co-located arrangements, paving the way to extensive acceptance and wide-spread adoption of co-located discretizations for unstructured grid solvers.

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0021-9991/$ – see front matter
http://dx.doi.org/10.1016/j.jcp.2013.11.006

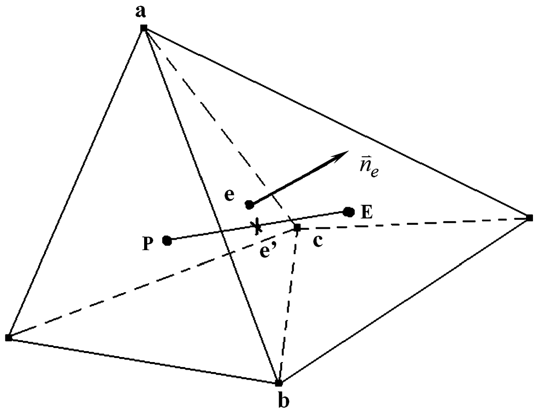

Fig. 1. Typical arbitrary discrete control volume (computational cell).

Despite its value and its wide-spread adoption, the Rhie–Chow interpolation technique contains several important defects that have been discovered and documented since its original publication. These defects include the following three: failure to suppress the checker-board decoupling for transient simulations with small time steps [7–10], dependence of the converged solution on the value of the under-relaxation factor [10–13], and its lack of robustness for problems with discontinuities [14] (whether the discontinuities arise from the flow solution itself, whether they are caused by discontinuities in material properties or the presence of porous regions in the flow field, or whether they are caused by discontinuous body forces [14,15]).

In this paper, the standard Rhie–Chow formulation is presented and the way in which it introduces dependence in the solution on the under-relaxation factor and, for transient problems, on the time-step size is analyzed. Further analysis is presented to show that the mass conservation error due to the Rhie–Chow interpolation is proportional to the square of the grid size multiplied with the fourth derivative of the pressure. This error remains small for smooth pressure fields but becomes appreciable in the presence of discontinuities in the pressure or the pressure gradient [14–16]. Discontinuities in the pressure or the pressure gradient arise, for example, in the following situations: buoyancy-driven flows in the vicinity of strong heat sources or large concentration gradients, multi-phase flows with large density ratios for the different phases, flows with strong swirl, flows with large surface tension forces, flows through resistive materials with discontinuities in the resistance, and flows through porous media with discontinuities in the porosity. In the case of porous media, discontinuities in the porosity cause discontinuities in the velocity field, and these in turn cause discontinuities in the pressure field.

Using the analysis presented in this paper, generalized formulations for the Rhie–Chow interpolation are developed to eliminate the three major defects of the standard Rhie–Chow interpolation and to extend the applicability of the standard Rhie–Chow interpolation to a greater range of flow conditions. These generalized formulations can be implemented in existing codes that use the Rhie–Chow interpolation to enhance their robustness and extend their ranges of applicability with a negligible increase in computational effort.

The generalized formulations derived in this work draw on several previous studies and modifications to the standard Rhie–Chow interpolation, including studies and modifications to eliminate the dependence of unsteady flow solutions on the time-step size [8–10], studies and modification to eliminate the dependence of converged solutions on the under-relaxation factor [10–12], and studies and modifications for more stable treatment of body forces [14,15]. The generalized formulations developed in this work for the Rhie–Chow interpolation eliminate all three of these defects in the standard formulation and also extend the formulation to large and rapidly varying body forces as well as to discontinuities in the flow field.

2. Discretization of the Navier–Stokes equations with the standard Rhie–Chow interpolation

Using a finite-volume discretization of the continuous Navier–Stokes equations, the momentum equation for the u component of velocity for cell P, such as the cell shown in Fig. 1, can be written in the following discrete algebraic form [4, 7]:

$$a_{u,P}(1+\alpha)u_P = \Big(\sum a_{u,nb}u_{nb}\Big)_P - V_P\left(\frac{\partial p}{\partial x}\right)_P + V_P B_{x,P} + V_P S_{x,P} + \frac{\rho_P V_P}{\Delta t}u_P^o + \alpha a_{u,P}u_P^* \tag{1}$$

$$a_{u,E}(1+\alpha)u_E = \Big(\sum a_{u,nb}u_{nb}\Big)_E - V_E\left(\frac{\partial p}{\partial x}\right)_E + V_E B_{x,E} + V_E S_{x,E} + \frac{\rho_E V_E}{\Delta t}u_E^o + \alpha a_{u,E}u_E^* \tag{2}$$