Journal of Computational Physics 255 (2013) 48–52

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Short note

Effects of heat conduction on artificial viscosity methods for shock capturing

Andrew W. Cook

Lawrence Livermore National Laboratory, Livermore, CA 94551, USA

ARTICLE INFO

Article history:
Received 18 December 2012
Accepted 1 August 2013
Available online 16 August 2013

Keywords:
Large-eddy simulation
Subgrid-scale models
Shocks
Turbulence
Artificial viscosity
Thermal conductivity
Stagnation enthalpy

ABSTRACT

We investigate the efficacy of artificial thermal conductivity for shock capturing. The conductivity model is derived from artificial bulk and shear viscosities, such that stagnation enthalpy remains constant across shocks. By thus fixing the Prandtl number, more physical shock profiles are obtained, only on a larger scale. The conductivity model does not contain any empirical constants. It increases the net dissipation of a computational algorithm but is found to better preserve symmetry and produce more robust solutions for strong-shock problems.

1. Introduction

Artificial viscosities have been employed for over six decades to capture shocks in high Mach number flows [1]. By incorporating viscous terms in the momentum and energy equations, shocks can be spread over several grid points, thus regularizing solutions on discrete meshes. At the molecular scale, shocks have internal structure, which depends not only on the viscosity but also on the thermal conductivity of the fluid. Typical Prandtl numbers for air and many other gases near atmospheric conditions are on the order of unity. However, in numerical simulations employing artificial viscosity, thermal conductivity is often neglected, or else employed in a manner unrelated to viscosity, such that the *effective* Prandtl number inside shocks is much greater than unity. This can have unintended consequences, such as “wall heating” [2,3]. The purpose of this Short note is to explore the pros and cons of an artificial conductivity that mimics the relationship of physical conductivity to physical viscosity. The artificial conductivity is designed to produce numerical shock profiles similar to physical shock profiles, but rescaled from the molecular realm to the grid scale.

2. Model derivation

For simplicity, consider a one-dimensional shock in a coordinate system in which the shock is stationary. The steady-state conservation equations can be spatially integrated to yield:

$$\rho u = \rho_1 u_1, \tag{1}$$

$$p + \rho u^2 - \tau_{xx} = p_1 + \rho_1 u_1^2, \tag{2}$$

$$\rho u\big(h + u^2/2\big) + q_x - u\tau_{xx} = \rho_1 u_1\big(h_1 + u_1^2/2\big), \tag{3}$$

E-mail address: awcook@llnl.gov.

0021-9991/$ – see front matter
http://dx.doi.org/10.1016/j.jcp.2013.08.003

where ρ is density, u is velocity, p is pressure, h is enthalpy, τ_{xx} is viscous stress, q_x is heat conduction and the 1 subscripts denote upstream supersonic conditions. The enthalpy is

$$h = e + p/\rho = c_p T \tag{4}$$

where e is thermal energy, T is temperature and c_p is constant-pressure specific heat. Eqs. (1), (2) and (3) apply locally within the shock wave. The Rankine–Hugoniot jump conditions require the stagnation enthalpy to match on either side of the shock; i.e.,

$$h_2 + u_2^2/2 = h_1 + u_1^2/2, \tag{5}$$

where the 2 subscripts denote downstream subsonic conditions. Comparison of (3) and (5) suggests that a useful form of artificial conductivity can be educed by requiring the stagnation enthalpy, $h + u^2/2$, to be constant inside the shock as well as on either side. This is equivalent to enforcing

$$\frac{dh}{dx} + u\frac{du}{dx} = 0 \tag{6}$$

throughout the shock wave. The Fourier heat flux then becomes

$$q_x \equiv -\kappa\frac{dT}{dx} = -\frac{\kappa}{c_p}\frac{dh}{dx} = \frac{\kappa}{c_p}u\frac{du}{dx}, \tag{7}$$

where κ is thermal conductivity. The Navier–Stokes viscous stress is

$$\tau_{xx} \equiv \left(\beta + \frac{4}{3}\mu\right)\frac{du}{dx}, \tag{8}$$

where β is bulk viscosity and μ is shear viscosity. We see from (3) that a constant stagnation enthalpy requires $q_x = u\tau_{xx}$ or $k/c_p = (\beta + 4\mu/3)$. Hence, a promising conductivity model for preserving monotonicity across the shock is

$$\kappa = (\beta + 4\mu/3)h/T. \tag{9}$$

A convenient feature of this model is that it does not involve any empirical constants. Note that for $\beta = 0$, (9) is equivalent to setting the Prandtl number to 3/4 [4]. Lee et al. [5] successfully employed (9) (with $\beta = 0$ and a temperature-dependent μ) in their Direct Numerical Simulations of a shock interacting with isotropic turbulence. Here we explore the efficacy of (9) for Large-Eddy Simulations (LES), wherein subgrid-scale models may be employed for μ and/or β.

3. Artificial viscosity

For the simulations reported herein, we employ the following grid-dependent viscosity models [6–10]:

$$\mu = \overline{0.002\rho\left|\nabla^4\left(SL^6\right)\right|}, \tag{10}$$

$$\beta = \overline{\rho\left|\nabla^4(\nabla\cdot\mathbf{u})\right|L^6 H(-\nabla\cdot\mathbf{u})}, \tag{11}$$

where S is the magnitude of the strain-rate tensor, L is the grid spacing, H is the Heaviside function, ∇^4 is the biharmonic operator and the overbar $\overline{()}$ denotes a Gaussian filter of width $4L$. The factor of 0.002 in (10) was empirically determined to produce the correct subgrid energy flux for decaying turbulence [7] and the Taylor–Green vortex [8]. The Navier–Stokes equations are solved in strong conservation form with spatial derivatives computed via tenth-order compact differencing [11] and temporal integration accomplished via fourth-order Runge–Kutta time-stepping [12]. Since the compact stencils are purely centered, there is zero numerical dissipation associated with the spatial differencing algorithm. The explicit Runge–Kutta time-stepping scheme introduces only very slight dissipation [6]. The role of μ and β in LES is to keep the solution smooth at the grid scale in order to avoid Gibbs phenomenon and other ringing associated with flow discontinuities.

4. Results

As a first test of the conductivity model (9), we consider the spherical Noh implosion [2]. The nondimensional initial conditions are: $\rho = 1$, $p = 0$ and $\mathbf{u}$ = unit vector directed toward origin, with an adiabatic index of $\gamma = 5/3$ (all test problems herein use an ideal-gas equation of state). In this problem, an infinite-strength shock expands outward from the origin at a constant radial velocity of 1/3. In Fig. 1, the results of simulations with and without thermal conductivity are compared to the exact solution. The artificial thermal conductivity is here seen to reduce wall heating and produce a shock slightly closer to the exact location. The conductivity model also reduces spurious oscillations behind the shock and helps preserve spherical symmetry.

As a second test of the artificial conductivity, we consider the spherical Sedov–Taylor–von Neumann blast wave [13–15]. Whereas the Noh problem is purely compressive, the Sedov blast wave is strongly expansive. The initial/flow conditions