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2D simulation of granular flow over irregular steep slopes using global and local coordinates

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ABSTRACT

In this work approximate augmented Riemann solvers are formulated providing appropriate numerical schemes for mathematical models of granular flow on irregular steep slopes. Fluxes and source terms are discretized to ensure steady state configurations including correct modeling of start/stop flow conditions, both in a global and a local system of coordinates. The weak solutions presented involve the effect of bed slope in pressure distribution and frictional effects by means of the adequate gravity acceleration components. The numerical solvers proposed are first tested against 1D cases with exact solution and then their results are compared with experimental data in order to check the suitability of the mathematical models described in this work. Comparisons between results provided when using global and local system of coordinates are presented. The obtained results point out that both the global and the local system of coordinates can be used to predict faithfully the overall behavior of the phenomena considered in this work.

1. Introduction

Landslides, rockfalls and debris avalanches take place when a mixture of mud, sand and rocks slide down a slope together. As suggested by Denlinger and Iverson [1] the study of granular flows constitutes a starting point for the understanding of the more complex mass movement phenomena mentioned before. This complexity is highlighted in the scientific community at the moment [2–6], in particular due to the strong effects of erosion processes or the presence of a fluid phase. As first approximation, several experiments on granular dry flows have been carried out in the past [7–10,5] as the initial target to be overcame by the numerical modeling tools.

Granular dry flows show fluid-like behavior where the front of the avalanche moves as a thin layer along high distances. Well known approaches for describing geophysical flows consider the Saint-Venant equations as a starting point. Depth averaged equations were first employed for solving geomorphologic flows by Savage and Hutter [7], where granular mass sliding was modeled including Coulomb-like basal frictions, and assuming a cohesionless Mohr–Coulomb type material. Since then, new mathematical models have appear in the literature. In [1] an extensive and complete review of predictive models for geomorphologic flows was provided and special attention was devoted to the computation of the Coulomb stresses conjugated to the deformation in solid-like behavior avalanches. Contrary to the Saint-Venant equations, defined in a Cartesian coordinate, the Savage–Hutter model uses a curvilinear coordinate along the topography. Denlinger and Iverson

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[1] formulated the depth-averaged governing equations referenced to a rectangular Cartesian coordinate system (with Z vertical) and in their new approach the estimated frictional stresses were defined with independence of the orientation of the coordinate system. The model was tested against analytical solutions and experimental data. Bouchut et al., [11] introduced an extra term in the original Savage–Hutter mathematical model, related to the curvature of the bottom, which is usually neglected when compared in terms of magnitude, in order to ensure the equilibrium at rest of the mass whatever the flow conditions (topography, friction coefficients, etc.). Some phenomenas, such as landslides, where the curvature terms play an important role can be found in [12,13]. In [14] this model was extended to consider an arbitrary coordinate system for shallow flow over a 2D topography, retaining the curvature terms. In recent works concerning geomorphologic flows over 2D irregular bed topographies [3,16] this term was omitted and promising computational results were obtained. Following [3,16] curvature terms related with the geometry are not considered here and the rheology of the material will be described using a Coulomb-type friction law.

Once a mathematical model is selected, another separate issue is the numerical scheme used. Considering the hyperbolic nature of the depth averaged equations, Godunov type schemes are commonly used in literature [1,17]. Godunov type schemes can be constructed departing from the definition of approximate solvers of the Riemann problem (RP). Approximate solvers provide a comprehensible definition of the conserved variables in the inner states of the same RP. Among the most successful and disseminated approximate solvers, Roe's method [18] and the HLL method [19], were defined to approximate solutions for hyperbolic system of equations without source terms. When including the presence of source terms in the system of equations, it is possible to extend the numerical schemes defined for the homogeneous case using point-wise explicit or implicit discretizations of the source terms. In [17], internal stresses represented by the Coulomb friction law were discretized using a point-wise implicit discretization. Numerical experimentation has shown that point-wise discretizations lead to undesirable results, as non-uniform discharge values in steady solutions [20,21]. A proper discretization of the frictional source terms must ensure a correct balance among fluxes and source terms (well-balanced property). Following previous work in well balanced numerical schemes for the shallow flow equations in presence of bed variations [22], in [1], the Roe scheme was applied in combination with an upwind technique applied to the internal stresses. The apparent topography method to deal with generic source terms in [14], based on the well-balanced property, was applied successfully to the simulation of the spreading of a granular column over a rough horizontal plane in [2] and over an inclined plane in [15]. Contrary to shallow water flows, where quiescent flow is given in cases of horizontal water level surface, in granular flows, steady state configurations include correct modeling of starting and stopping flow conditions [14,15,3,16].

The presence of source terms leads to non-strictly hyperbolic systems of equations and, as a consequence, they have an impact in the solution of the RP. In the shallow water equations with variable topography, different approximations to the Riemann problem have been presented in the literature [23–28]. The properties of these RP solutions not only must guarantee the well-balanced property, but also, the associated numerical scheme must ensure convergence to the solution. Convergence to the solution is not an easy task, as in problems with source terms the total number of waves can be larger than the number of characteristic fields [28]. In the search of approximate solvers linked to the presence of source terms, two augmented solvers named ARoe (Augmented Roe) and HLLCS (HLL with Contact wave and Source terms) were presented in [29] and [30] respectively. The approximate solver in [29] was based on the upwind discretization of the source terms in [31] and the Roe solver defined for the homogeneous case. In [30] the HLLCS was constructed by including the presence of source terms in the HLLC [32] solver. Both augmented solvers were constructed by including an extra wave associated to the presence of source terms, providing a complete description of the evolution of the conserved variables in the inner estates of the RP. In consequence, the Godunov-type schemes developed were able to avoid the appearance of instabilities and negative values of the flow depth in presence of source terms. The ARoe solver was exploited in [33] allowing correct approximate solutions of wave Riemann problems involving complex rheology and complex geometry when using depth average equations. An accurate and robust first order finite volume scheme, able to handle correctly transient problems including modeling of starting and stopping flow conditions was presented in [33]. Then, in contrast with prior works, [14,3], where numerical fluxes were constructed to ensure well-balanced arguments, in [33], the definition of the complete approximate solution ensured correct integral estimations of the source terms under all type unsteady of flow conditions. It is worth mentioning, that, in general, only in cases of quiescent equilibrium, the source terms can be integrated exactly. In any other case, the approximate solver provides the rules to avoid unphysical results, allowing the correction of the estimations made for the source terms if necessary. This result is of utmost importance when modeling of starting and stopping flow conditions and can be applied with independence of the type of rheological model selected.

In presence of steep slopes the usual hypothesis of hydrostatic pressure in the vertical direction Z in the shallow water equations is not longer admissible. This fact has consequences when deriving the mathematical model. In global coordinates (X, Y, Z) (Fig. 1), the gravity vector has a simple form

$$\mathbf{g} = -(g_X, g_Y, g_Z)^T = (0, 0, -g)^T \tag{1}$$

and the bed vector $\mathbf{S}_o$ can be written as

$$\mathbf{S}_o = (\tan\theta, \tan\gamma) = \left(-\frac{\partial Z_b}{\partial X}, -\frac{\partial Z_b}{\partial Y}\right) \tag{2}$$