Journal of Computational Physics 255 (2013) 228–244

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

A hybrid level set–front tracking finite element approach for fluid–structure interaction and two-phase flow applications

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ARTICLE INFO

Article history:
Received 4 October 2012
Received in revised form 22 June 2013
Accepted 5 August 2013
Available online 28 August 2013

Keywords:
Level set method
Front tracking
Mesh optimization
Arbitrary Lagrangian–Eulerian formulation
Fluid–structure interaction
Two-phase flow

ABSTRACT

We present a hybrid level set–front tracking approach suitable for fluid–structure interaction and two-phase flow applications. Our approach aims at extending geometrical flexibility of standard mesh moving/front tracking methods by introducing an additional implicit level set representation of the geometry under consideration. The computational mesh is automatically aligned to the implicitly described geometry by minimizing a nonlinear, constrained functional. Resulting triangulations approximate the geometry accurately while being optimal in a certain sense. Due to the mesh alignment, finite element spaces defined on these triangulations may be easily adjusted to account for special solution properties such as discontinuities across interfaces. In order to demonstrate the flexibility of the proposed approach, we apply it to a simplified one-way coupled fluid–structure interaction problem inspired by the flow induced by a moving cardiac valve. Furthermore we evaluate the approach by solving a two-phase flow benchmark problem.

1. Introduction

Sharp interfaces form an essential part of many numerical models used in computational studies of multiphysics applications. These interfaces may emerge from modeling the interaction between fluids and structures or other fluids. Let us just mention a few application scenarios: fluid–structure interaction (FSI) problems include the blood flow in the heart interacting with heart valves [29,43–45] or flow induced vibrations of airfoils [39,10]. In both cases a sharp interface separates the fluid from a solid structure. When simulating multiphase flows, immiscible fluids may be assumed to be separated by a sharp interface. As an example consider the dynamics of individual gas bubbles or swarms of gas bubbles rising in a viscous liquid [5,34].

When it comes to formulating a numerical method, the representation of interfaces or boundaries plays a decisive role. An overview of numerical methods for tracking interfaces can be found in [18]. We propose a hybrid method based on an explicit front tracking method and an additional implicit, level set based representation of the interface. Therefore, we briefly recapitulate some variants of each methodology and corresponding applications. We should mention that the following overview is by no means complete, but is rather meant to emphasize the intermediate nature of the proposed framework.

Explicit tracking methods have been widely used to describe evolving interfaces. Unverdi and Tryggvason [42] simulated multiphase flows in a front capturing manner by using a fixed, stationary mesh for the flow field and an additional, separate, unstructured mesh for the representation of the interface. A marker point approach to tracking the interface for various applications such as bubbly flows, atomization and solidification was proposed in [40]. Lagrangian methods using

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0021-9991/$ – see front matter
http://dx.doi.org/10.1016/j.jcp.2013.08.018

computational meshes which follow the motion of the interface by using an Arbitrary Lagrangian–Eulerian (ALE [17]) formulation of the model under consideration are common in many different applications. The movement of the mesh may be taken into account either by using conservative interpolation techniques (rezoning/remapping, see for instance [11,22]) or by directly solving the equations in a moving coordinate system. Hu et al. [16] used such a monolithic ALE technique for a direct numerical simulation of fluid–solid systems (ALE particle mover) and Bänsch et al. [3] solved a coupled Navier–Stokes equations/Stefan problem modeling the partial melting process of a metal workpiece. Although these methods offer many advantages provided by the explicit representation of the interface, problems arise whenever strong deformations or even topological changes of the interface lead to a degeneration of the computational mesh. In the latter two applications [16,3] remeshing techniques to overcome mesh distortion were proposed. However, remeshing usually leads to an additional source of errors since quantities of interest have to be transferred from the old mesh to the new mesh.

In the level set method [28,27], the interface is described implicitly by the zero level set of an additional function ϕ. The motion of the interface then usually is accounted for by solving an additional convection equation for ϕ (the level set equation). By describing the interface implicitly, level set methods are able to deal with strong deformations of the interface intrinsically without the danger of mesh distortion. Moreover, interfacial quantities such as curvature or normal vectors of the interface are readily obtained from ϕ. However, the purely hyperbolic character of the level set equation and the lack of mass conservation in the level set equation require additional care. There exist a lot of different methods and techniques which deal with these issues. Let us just mention the combined level set/volume-of-fluid method (CLSVOF [36,37]) as an example, which aims at combining the advantageous mass conservation properties of volume-of-fluid methods with the advantages of the level set formulation mentioned above.

While in level set methods the interface often is kept implicit it turns out that in many scenarios an explicit representation of the interface may be advantageous. For instance when modeling multiphase flows with surface tension, a jump of the pressure across the interface needs to be taken into account. In the context of finite elements, this jump may be captured by introducing additional discontinuous basis functions at the interface yielding the extended finite element method (X-FEM [35,6,13]). However, since the mesh does generally not conform to the interface, special reconstruction and quadrature techniques have to be considered. A mesh aligned to the interface would allow for a much simpler introduction of finite element spaces which automatically account for problem-specific properties such as discontinuities.

For this reason, several attempts have been made to capture the zero level set explicitly by the computational mesh. Li and Shopple [21] designed a method to obtain an interface fitted mesh from a fixed base mesh by introducing new nodes at the zero level set. Several level set related techniques such as reinitialization and a special curvature approximation through the level set function are considered. The resulting method is able to handle complex solvation and solidification applications. Nochetto and Walker [24] introduced a hybrid variational front tracking/level set mesh generator. Their technique makes use of remeshing, local adaptivity and mesh smoothing, which enables them to handle problems exhibiting large deformations and topological changes.

The methods mentioned so far rely on either completely changing computational meshes or, at least, changing the topology of the mesh by inserting new nodes and elements. Ohtake et al. [25,26] proposed an iterative method which, given an implicit level set description of the interface and an initial polygonization, optimizes the position of the vertices located at the interface. Vertices of the resulting triangulations are aligned to the zero level set. The induced normals of the polygonization yield the best approximation of normal vectors implicitly given by ϕ.

Our method is unique in the sense that one fixed base mesh is adapted to the implicit representation of the interface while maintaining mesh connectivity. The proposed variational approach guarantees mesh optimality and does not rely on any combinatorial considerations prevalent in common level set reconstruction techniques. The resulting explicit representation of the interface allows for a simple definition and efficient implementation of problem-specific finite element spaces. Due to the fixed mesh connectivity, an existing ALE code can be augmented easily by the proposed method, providing it with enhanced geometrical flexibility.

The rest of this paper is organized as follows. In Section 2 we briefly revisit the fundamental building block of our mesh alignment technique based on a variational approach to mesh optimization. Section 3 is concerned with the alignment of a given computational mesh to the geometry implicitly described by a level set function. The proposed approach results in a nonlinear, constrained optimization problem. A simple quadratic penalty method is presented to solve the optimization problem. We quantify the approximation quality of the discrete interface using a priori estimates and performing convergence studies. Different strategies to further increase the approximation quality by using isoparametric finite elements are considered.

Equipped with a method aligning the mesh to a given zero level set, we discuss its integration in an existing ALE framework in Section 4. Furthermore, we present the mathematical model for a simplified fluid–solid interaction problem which cannot be handled by classical ALE front tracking methods without problem-specific extensions. With respect to modeling two-phase flows, the mesh alignment together with an isoparametric representation of the interface is advantageous in the definition of finite element spaces able to capture discontinuities at the interface.

Numerical results can be found in Section 5. In order to show the advantage of having an explicit representation of the interface in the hybrid approach and the usage of adapted finite element spaces, we first consider a stationary droplet with surface tension. The FSI example is inspired by the movement of a cardiac valve inducing blood flow. This example emphasizes the enhanced geometrical flexibility arising from the proposed hybrid framework. Finally, we apply the method to a fully coupled two-phase flow benchmark problem and compare the results to a reference solution.