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A closure-independent Generalized Roe solver for free-surface, two-phase flows over mobile bed

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ABSTRACT

Several different natural phenomena can be studied in the framework of free-surface, two-phase flows over mobile bed. Mathematically, they can be described by the same set of highly nonlinear, hyperbolic nonconservative PDEs but they differ in the possible algebraic closure relations. These affect significantly the relevant eigenvalues and consequently, all finite-volume numerical methods based on upwind Godunov-type fluxes. In this work the Generalized Roe solver, introduced in [29] for the case of a specific closure, is reformulated in a complete closure-independent way. This gives the solver a quite general applicability to the class of problems previously mentioned. Moreover, the new method maintains all the desirable features shown by the original one: full two-dimensionality and exact well-balanceness. This result is made possible thanks to the development of a novel Multiple Averages (MAs) approach that allows a straightforward determination of the matrices required by the solver. Several tests show the capabilities of the proposed numerical strategy.

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1. Introduction

Nature presents several flows in which a mixture of relatively small solid particles move inside a fluid forming a free-surface, two-phase mixture flow. This happens in rivers, where the volume concentration of sediments is commonly low and the process is called sediment-transport. Here, phase interaction is weak and the sediments are essentially driven by the water flow. Also debris-flows are two-phase flows. They are characterized by high concentration of sediments and strong phase interaction. Therefore, stress and velocity distributions are quite different from the case of sediment transport [2]. Finally, some types of snow avalanches are two-phase flows: here the solid phase is composed by a granular structure of snowflakes while the fluid phase can be composed by liquid or air. Obviously, the distributions of stress and velocity are different from the previous two cases.

Despite the differences, these flows can be described within a unified framework of a highly nonlinear set of non-conservative partial differential equations. The mathematical description of such flows is commonly based on a continuum approach: properties that are defined only inside a given phase (e.g. the solid density is defined inside the particles) through a proper averaging process become phase-averaged properties defined in any point of the space. Averaging can be made over the volume (e.g. [16]) or over an ensemble of configurations (e.g. [31]). Then, by averaging the conservation principles of mass and momentum for the liquid and the solid phase, the phase-averaged system of equations for the considered mixture is obtained. The solid phase thus becomes a continuum with behavior similar to a fluid (granular fluid). Following

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a different approach (e.g. [17]) it is also possible to write the governing equations of a two-phase flow in a more axiomatic way without deriving them from an averaging process.

In the present work we are interested in a two-dimensional formulation of the problem since, from a practical point of view, in all the above mentioned flows the characteristic depth-scale is much smaller than the areal extension-scale. Moreover, we are interested in flows where a lower mobile interface divides the flowing particles from the idle ones. This interface is commonly called "bed". Since its position changes with time according to the flow conditions, these flows are called mobile-bed ones. This feature distinguishes the present approach from the majority of works devoted to two-phase granular flows where fixed-bed conditions are commonly considered (among others, [22–24]). Finally, in low concentration regimes sediments may deviate from the depth-averaged velocity (see e.g. [15]). In high concentration regimes this has not yet been observed experimentally, but theoretical investigations suggest that it could occur. Anyhow, following [29], this phenomenon can be included in a diffusive term that can be superimposed to the basic assumption that the velocity of the two phases coincides, both in modulus and direction (isokinetic approach). In this work we are interested only in the advective part of the flow and so we disregard all possible diffusive terms. The resulting system is then composed by three conservation equations: two of them are scalar, namely the solid and the liquid mass conservation, and one is vectorial, i.e. the linear momentum conservation of the bulk mixture.

Under the previous assumptions, and referring to Fig. 1, the resulting system of equations can be written as [3]:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}_x}{\partial x} + \frac{\partial \mathbf{F}_y}{\partial y} + \mathbf{H}_x \frac{\partial \mathbf{W}}{\partial x} + \mathbf{H}_y \frac{\partial \mathbf{W}}{\partial y} = \mathbf{T} \tag{1}$$

where $\mathbf{U}$ is the vector of the conservative variables, $\mathbf{F}_x$, $\mathbf{F}_y$ are the conservative fluxes and $\mathbf{H}_x \partial \mathbf{W}/\partial x$, $\mathbf{H}_y \partial \mathbf{W}/\partial y$ are the nonconservative ones in x and y directions respectively. These quantities can be expressed as a function of the volume concentration of the solid phase c and of the primitive variable vector $\mathbf{W} = [h, u_x, u_y, z]^T$, where h is the flow depth, $\mathbf{u} = (u_x, u_y)$ is the vector of the depth-averaged mixture velocity and z is the elevation of the bed respect to a horizontal reference plane. Finally, $\mathbf{T}$ is the bed stress vector. The detailed expression of each of these terms will be provided in Section 2.

The total number of unknowns for a problem of this type is seven: four components of the vector $\mathbf{W}$, the two non-null components of $\mathbf{T}$ plus the concentration c. However, the number of equations is only four and therefore three closure relations are needed. Usually, the concentration and the stress terms are expressed as algebraic functions of the primitive variables. It is exactly at this level that the system of equations becomes specific to the type of flow that has to be modeled: different flows need different closure relations. Moreover, within the same flow type, several different empirical, semi-empirical or theoretical expressions are available in the literature. This is essentially due to a certain lack of knowledge that is still present in this field. In any case, what is important for this work is to notice that the closure adopted for the concentration affects significantly the eigenstructure of the problem and this, in turn, has a strong impact on the numerical schemes we are interested in, i.e. finite-volume methods with upwind Godunov-type fluxes. In fact, whenever the closure relation is changed, all the expressions for evaluating eigenvalues and eigenvectors must be changed accordingly as well as the relevant numerical scheme. Furthermore, in several cases the closure is such that an explicit analytical expression of the eigenstructure is too complex to be worked out. Remaining in the framework of upwind methods, a possible approach is developing a numerical scheme valid for a quite simplified relation and then using it to approximate locally a more complex relation (see. e.g. [19]). Nevertheless, more general and accurate approaches are desirable.

Another feature of system (1) that affects the mentioned numerical schemes, is the presence of nonconservative terms. From a numerical point of view, this leads to the problem of well-balancing the flux terms with the nonconservative ones in steady states without source terms. In particular, an exact well-balanced scheme is highly desirable but, depending on the closure adopted, this may not be an easy matter [11,12]. An example of a well-balanced scheme designed to deal with the nonconservative terms described above is the Generalized Roe (GR) solver proposed by [27] in the context of a 1D finite-volume numerical method for the problem (1). In [29] a two-dimensional GR solver is incorporated into a finite-volume debris-flow model and its performance is tested with good results. Still, the derivation of the matrices to be used in the solvers, depends in both papers on the adopted closure.

The main target of this work is to develop a new robust closure-independent GR solver that maintains all the desirable properties shown in [29] for the case of a specific closure: full two-dimensionality and exact well-balanceness. Two main ingredients are necessary to achieve the goal: (1) a simple and systematic procedure to obtain the matrices used in the solver, (2) a suitable generalization of the previous procedure in order to allow the use of a generic closure relation. The first task has been faced developing and generalizing an idea already used in [27,29]: instead of looking for a single averaged state, to be evaluated in a proper way starting from the left and right states of a local Riemann Problem (RP), the Jacobian matrices used in the GR solver are computed considering a suitable set of averages (hence the name of Multiple Averages, hereafter MAs). The second task has been tackled using a generic functional expression as closure equation, without specifying its actual value until the end of the procedure. The details of these two elements are given further in the paper. Here it is sufficient to say that the resulting solver is completely general and can be even used with complex closures written in implicit form. In this way we have provided a general, powerful and accurate tool for solving a wide range of two-phase, free-surface problems.

The structure of the paper is the following. In Section 2 we briefly present some features of the mathematical model. In Section 3 we present the MAs methodology as a general approach to obtain a well-balanced Generalized Roe solver.

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