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Trust-region based solver for nonlinear transport in heterogeneous porous media

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ABSTRACT

We describe a new nonlinear solver for immiscible two-phase transport in porous media, where viscous, buoyancy, and capillary forces are significant. The flux (fractional flow) function, F, is a nonlinear function of saturation and typically has inflection points and can be non-monotonic. The non-convexity and non-monotonicity of F are major sources of difficulty for nonlinear solvers of coupled multiphase flow and transport in natural porous media. We describe a modified Newton algorithm that employs trust regions of the flux function to guide the Newton iterations. The flux function is divided into saturation trust regions delineated by the inflection, unit-flux, and end points. The updates are performed such that two successive iterations cannot cross any trust-region boundary. If a crossing is detected, the saturation value is chopped back to the appropriate trust-region boundary. The proposed trust-region Newton solver, which is demonstrated across the parameter space of viscous, buoyancy and capillary effects, is a significant extension of the inflection-point strategy of Jenny et al. (JCP, 2009) [5] for viscous dominated flows.

We analyze the discrete nonlinear transport equation obtained using finite-volume discretization with phase-based upstream weighting. Then, we prove convergence of the trust-region Newton method irrespective of the timestep size for single-cell problems. Numerical results across the full range of the parameter space of viscous, gravity and capillary forces indicate that our trust-region scheme is unconditionally convergent for 1D transport. That is, for a given choice of timestep size, the unique discrete solution is found independently of the initial guess. For problems dominated by buoyancy and capillarity, the trust-region Newton solver overcomes the often severe limits on timestep size associated with existing methods. To validate the effectiveness of the new nonlinear solver for large reservoir models with strong heterogeneity, we compare it with state-of-the-art nonlinear solvers for two-phase flow and transport using the SPE 10 model. Compared with existing nonlinear solvers, our trust-region solver results in superior convergence performance and achieves reduction in the total Newton iterations by more than an order of magnitude together with a corresponding reduction in the overall computational cost.

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1. Introduction

Accurate numerical modeling of immiscible multiphase flow in subsurface porous formations is used to manage subsurface resources, including oil reservoirs and water aquifers. Moreover, numerical flow simulation in natural porous media is critical for the design and management of subsurface CO_2 sequestration projects. The saturation equations, which describe

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the transport of the fluid phases in space and time, are highly nonlinear [1,2]. They are characterized by non-convex flux functions that can also be non-monotonic in the presence of strong buoyancy forces [3–5].

The use of explicit time integration schemes for the saturation equations of large-scale reservoir models often leads to severe restriction on the timestep size. This is mainly due to the combination of strong permeability heterogeneity and non-convex flux functions. In reservoir simulation problems of large heterogeneous domains, where the evolution of the saturation field is sought, it is often the case that for a given global timestep size, the corresponding Courant–Friedrichs–Lewy (CFL) numbers can vary by orders of magnitude across the computational domain [6]. In such cases, the use of explicit time integration schemes is not feasible, and implicit time integration is required. The backward Euler scheme, which is referred to as the Fully Implicit Method (FIM) [1], is often combined with phase-based upstream weighting. The FIM formulation yields a system of nonlinear discrete conservation equations, one equation for each of the immiscible phases. The nonlinear system is usually cast in residual form and solved using the Newton method, whereby a sequence of iterations, each iteration involving the construction of the Jacobian matrix and solution of the resulting linear system, is performed until the solution of the coupled nonlinear algebraic equations is obtained for the target timestep. The FIM scheme is unconditionally stable, but there is no guarantee that the Newton solver will converge [1]. In reservoir simulation practice, heuristic techniques are usually employed to control the timestep size during the simulation run [7–12]. The use of such heuristics often leads to timestep sizes that are too conservative resulting in unacceptably large computational time and wasted computations [13]. Our objective is to develop a nonlinear solver for coupled multiphase flow and transport in heterogeneous porous media that is unconditionally convergent, so that the timestep size can be chosen based solely on accuracy considerations without worrying about the robustness of the nonlinear solver itself. An important step in this direction was taken by Jenny et al. [5], who provided strong evidence that the nonlinearity of the coupled conservation laws is dominated by the nonlinearity of the flux function. They described a simple chopping strategy of the Newton updates that is guided by the properties of the flux function.

In this work, we take a significant step further. We describe a nonlinear Newton-based solver that allows for taking arbitrarily large time steps across the entire viscous-buoyancy-capillary parameter space of immiscible two-phase transport in porous media. The mathematical model is introduced in Section 2. Section 3 describes previous work of direct relevance to our proposed solution strategy. In Section 4, an unconditionally convergent nonlinear solver for one-dimensional transport across the entire viscous-gravity-capillary parameter space is described. We also report and discuss detailed numerical simulations of coupled two-phase flow in highly heterogeneous 3D domains. Finally, in Section 5 we present our conclusions.

2. Mathematical model

We consider nonlinear immiscible, incompressible, two-phase flow in porous media. The conservation law for the two phases – referred to as wetting and nonwetting – can be written as [1,2,4]:

$$\phi\frac{\partial S_w}{\partial t} + \nabla\cdot\mathbf{u_w} = q_w, \tag{1}$$

$$\phi\frac{\partial S_n}{\partial t} + \nabla\cdot\mathbf{u_n} = q_n, \tag{2}$$

where ϕ is the porosity of the medium. We use subscript α to denote the phases, i.e., w or n. S_α is the saturation, $\mathbf{u}_\alpha$ is the velocity, and q_α is the source term. The phase velocity is given by Darcy's law:

$$\mathbf{u}_\alpha = -k\frac{k_{r\alpha}}{\mu_\alpha}(\nabla p_\alpha + \rho_\alpha g\nabla h), \quad \alpha = w, n \tag{3}$$

where p_α is the pressure, ρ_α is the density, h is the height, $k_{r\alpha} = k_{r\alpha}(S_\alpha)$ is the relative permeability, and μ_α is the viscosity.

Substitution of Eq. (3) into Eqs. (1) and (2) yields a coupled system of nonlinear parabolic equations. The system often exhibits a mixed elliptic–hyperbolic character, which becomes apparent when we sum up Eqns. (1) and (2) to obtain the pressure equation.

$$\nabla\cdot\big(-k\lambda_t\nabla p_w - kg(\lambda_w\rho_w + \lambda_n\rho_n)\nabla h - k\lambda_n\nabla p_c\big) = q_t, \tag{4}$$

where $p_c = p_c(S_w)$ is the capillary-pressure versus saturation relationship, and the total mobility $\lambda_t = \lambda_w + \lambda_n$.

We can rewrite the wetting phase velocity in terms of the total-velocity, $\mathbf{u_t} = \mathbf{u_w} + \mathbf{u_n}$, as

$$\mathbf{u_w} = \frac{\lambda_w}{\lambda_t}\mathbf{u_t} - kg\frac{\lambda_w\lambda_n}{\lambda_t}(\rho_w - \rho_n)\nabla h + k\frac{\lambda_w\lambda_n}{\lambda_t}\nabla p_c. \tag{5}$$

Substituting Eq. (5) into Eq. (1), the transport (saturation) equation is obtained as:

$$\phi\frac{\partial S_w}{\partial t} + \nabla\cdot\left(\frac{\lambda_w}{\lambda_t}\mathbf{u_t} - kg\frac{\lambda_w\lambda_n}{\lambda_t}(\rho_w - \rho_n)\nabla h + k\frac{\lambda_w\lambda_n}{\lambda_t}\nabla p_c\right) = 0. \tag{6}$$

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