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Circular function-based gas-kinetic scheme for simulation of inviscid compressible flows

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ABSTRACT

This paper presents a new gas-kinetic scheme for simulation of compressible inviscid flows. It starts to simplify the integral domain of Maxwellian distribution function over the phase velocity ξ and phase energy ζ to the integral domain of modified Maxwellian function over the phase velocity ξ only. The influence of integral over phase energy ζ is embodied as the particle internal energy e_p. The modified Maxwellian function is further simplified to a circular function with the assumption that all the particles are concentrated on a circle. Then two circular function-based gas-kinetic schemes are presented for simulation of compressible inviscid flows. In the new schemes, no error and exponential functions, which are often appeared in the Maxwellian function-based gas-kinetic schemes, are involved. As a result, the new schemes can be implemented in a more efficient way. To validate the proposed new gas-kinetic schemes, test examples in the transonic flow, supersonic flow and hypersonic flow regimes are solved. Numerical results showed that the solution accuracy of the circular function-based gas-kinetic schemes is comparable to that of corresponding Maxwellian function-based gas-kinetic schemes. However, the circular function-based gas-kinetic schemes need less computational effort.

1. Introduction

In the last thirty years, the gas-kinetic scheme has been developed into an attractive approach for simulation of fluid flows in a wide range. The approach is usually based on the solution of Boltzmann equation and Maxwellian distribution function. It computes the inviscid and viscous fluxes as a single entity, which is more appealing than conventional Navier–Stokes solvers. Due to this feature, it attracts more and more attention from researchers [1–5].

The early development of gas-kinetic scheme can be referred to the work of Pullin [6], in which the equilibrium flux method (EFM) was proposed for simulation of inviscid flows. EFM splits the Maxwellian function into two parts and the complete error function is used to calculate numerical fluxes. By solving collisionless Boltzmann equation, Mandal and Deshpande [7] also derived the same scheme, which is named as Kinetic Flux Vector Splitting (KFVS) scheme. Since the split fluxes of KFVS scheme are smooth functions of the Mach number, no special effort is needed to ensure the smooth behavior of split fluxes. Subsequently, Chou and Baganoff [8] further developed the KFVS scheme for the simulation of viscous flows. The positivity of the first-order KFVS scheme can be proven rigorously [7,9]. This suggests that the pressure, density and internal energy can be kept positive in every control volume and at each time step for simulation of compressible flows. Although it has some unique features, the KFVS scheme usually gives poorer results than those obtained from Godunov

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or Flux Difference Splitting (FDS) schemes [10,11]. The cause may be that numerical dissipation of the KFVS scheme is proportional to the mesh size [12]. Only for those cases where the physical viscosity is larger than the numerical viscosity, the KFVS scheme could give accurate Navier–Stokes solutions.

To improve the solution accuracy of the KFVS scheme, the so-called Bhatnagar–Gross–Krook (BGK) gas-kinetic scheme was developed by Prendergast and Xu [13], Chae et al. [14], Xu [15] and other researchers. Unlike KFVS scheme, the BGK scheme considers the particle collisions during the gas evolution stage and a BGK collision model [16] is applied in the flux evaluation. As a result, the dissipation in the streaming process is controlled by the collision time rather than by the time step. Numerical results showed that the BGK scheme can accurately simulate both inviscid and viscous flows [17,18]. It should be noted that Xu and his co-workers [1,3,9,12,13,15,19,20] have made a great contribution in the development and application of BGK gas-kinetic scheme.

As far as we know, most of existing gas-kinetic schemes are based on the Maxwellian function. Due to complication of Maxwellian function, the gas-kinetic schemes are usually complicated and need more computational effort. This may bring some difficulties for new users to implement the schemes. In order to obtain a simple and efficient scheme, two circular function-based gas-kinetic schemes are presented in this work. In the present schemes, the original Maxwellian function, which is the function of phase velocity ξ and phase energy ζ, is firstly simplified into the function of phase velocity ξ only. The effect of phase energy ζ is embodied as the particle internal energy e_p. Then, the simplified Maxwellian function is further reduced to a circular function with assumption that all the particles are concentrated on a circle. In this way, the integral in the infinite domain of phase velocity ξ and phase energy ζ is reduced to the line integral along the circle. By using the circular function, a circular function-based KFVS (CKFVS) scheme and a circular function-based BGK (CBGK) scheme are proposed for simulation of compressible inviscid flows. Since no error and exponential functions are involved in the evaluation of numerical fluxes, the present schemes are more efficient than the corresponding Maxwellian function-based gas-kinetic schemes. To validate the developed schemes, some test cases, such as regular shock reflection, double Mach reflection, the diffraction of a supersonic shock moving around a 90° corner, compressible flow around RAE2822 airfoil and hypersonic flow around half cylinder, are simulated. As compared with Maxwellian function-based KFVS (MKFVS) scheme and BGK (MBGK) scheme, the computational efficiency of present schemes is improved by about 60%. In the meantime, the solution accuracy of present schemes is comparable to that of Maxwellian function-based gas-kinetic schemes.

2. Methodology

2.1. Particle potential energy is independent of phase velocity

The continuous Boltzmann equation with Bhatnagar–Gross–Krook (BGK) collision model [16] can be written as

$$\frac{\partial f}{\partial t} + \xi \cdot \nabla f = \frac{g - f}{\tau} \tag{1}$$

where f is the gas distribution function and g is the equilibrium state approached by f through particle collisions within a collision time scale τ. Both f and g are functions of space, time, phase velocities (ξ_i, $i = 1, \ldots, D$) and phase energy (ζ_j, $j = 1, \ldots, K$). The equilibrium state is a Maxwellian function

$$g = g_M = \rho\left(\frac{\lambda}{\pi}\right)^{\frac{D+K}{2}} e^{-\lambda[\sum_{i=1}^{D}(\xi_i - U_i)^2 + \sum_{j=1}^{K}\zeta_j^2]} \tag{2}$$

Here, ρ is the density of mean flow, U_i is the macroscopic flow velocity in the i-direction of D-dimensional space, and $\lambda = m/(2kT) = 1/(2RT)$, where m is the molecular mass, k is the Boltzmann constant, R is the gas constant and T is the temperature. The number of phase energy variables is $K = 3 - D + N$, where D is the abbreviation of dimension and N represents the number of rotational degrees of freedom. The specific heat ratio γ can be expressed as

$$\gamma = \frac{b+2}{b} = \frac{K + D + 2}{K + D} \tag{3}$$

where, b is the total number of degrees of freedom of molecules. For monatomic gas and diatomic gas, b is taken as 3 and 5 respectively.

With Maxwellian function (2), to recover Euler equation by Eq. (1) through Chapman–Enskog expansion analysis, the following 5 conservation forms of moments have to be satisfied,

$$\int g_M \, d\Xi = \rho \tag{4a}$$

$$\int g_M \xi_\alpha \, d\Xi = \rho u_\alpha \tag{4b}$$

$$\int g_M \left(\xi_\alpha \xi_\alpha + \sum_{j=1}^{K} \zeta_j^2\right) d\Xi = \rho(u_\alpha u_\alpha + bRT) \tag{4c}$$