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Simulations of moist convection by a variational multiscale stabilized finite element method

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ABSTRACT

A variational multiscale stabilized finite element scheme is presented for the solution of moist atmospheric flows. The fully compressible Euler equations are coupled to a system of three advection equations that model the transport of water quantities in the atmosphere. A Kessler-type parametrization of microphysical processes of warm rain is used. Because analytic solutions to this problem are not available, the model is assessed by comparison with similar simulations presented in the literature. The metrics for evaluation are the intensity and spatial distribution of the storm, its duration, the location of precipitation, and water accumulation at different grid resolutions. The current model is able to capture the principal features of two-dimensional convective storms and orographic clouds at the grid scales typical of mesoscale atmospheric simulations.

1. Introduction

Because of computational issues on massively parallel architectures and the increasing interest for grid adaptivity in atmospheric problems, alternative methods to Finite Differences (FD) have been extensively applied to Numerical Weather Prediction (NWP). On the one hand Finite Volumes (FV), and on the other hand element-based methods such as Discontinuous Galerkin (DG), Finite Elements (FE), and Spectral Elements (SE). In this respect, two philosophies can be distinguished: high-order methods by SE and DG [1–6], or low order methods using FV [7–9] and FE [10]. The finite element method has seen a wide spread use in different fields of computational mechanics since its first appearance in the 50s. In atmospheric simulations the finite element method dates back to the 70s starting with [11], among others, and has continued ever since (see, e.g. [12–15]).

In [16], we first used the variational multiscale stabilized finite element method (FE VMS) to solve the compressible Euler equations for stratified non-hydrostatic flows of dry atmospheres. In this paper, we assess the ability to simulate two-dimensional idealized moist dynamics. The Euler equations are coupled to a set of prognostic equations for water species that, by means of the Kessler microphysics scheme [17], are allowed to change phase and trigger a long-lasting two-dimensional convective storm. Water in the atmosphere is present in different forms and phases that may coexist simultaneously. We are interested in the simplest of water species interactions; in particular, the microphysical processes that drive condensation of water vapor into clouds and formation of rain. In the absence of solid water (ice, snow, graupel, hail)

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one talks about *warm* microphysics, where a simple bulk parametrization can predict cloud water and rain/drizzle only. Bulk models assume that the number of water categories in the atmosphere can be grouped such that as few species as possible need to be solved explicitly [18]. Due to the well known limitations of two-dimensional cloud modeling [19–22], in this study we do not expect to represent faithfully the most proper characteristics of convective storms. Nevertheless, two-dimensional simulations can be used to verify the properties of the FE VMS algorithm for moist atmospheric simulations. The literature is sufficiently generous in the presentation of two-dimensional storm analysis to compare against [23–26]. An approach similar to that of [23] is used for the analysis and assessment of the algorithm presented in this paper. Two tests are performed: (i) simulation of a convective storm triggered by a rising thermal perturbation, and (ii) simulation of an orographic precipitating cloud generated by purely mechanical motion of air aloft a mountain.

The remainder of the paper is organized as follows. Some notation and the basic thermodynamics of a moist atmosphere are introduced in Section 2. The equations are presented in Section 3. The structure of the algorithm and the numerical method are reported in Sections 4 and 5, respectively. In Section 6 the numerical tests are presented, mass conservation is discussed in Section 7, and conclusions are given in Section 8.

2. Microphysics and definitions for moist atmospheres

Cloud microphysics includes all the thermo-physical processes at the scales of the particles that form the cloud. Examples are the phase change of water quantities or the agglomeration of particles into larger ones. Most physical processes typical of storm dynamics (e.g., precipitation, freezing, deposition, or sublimation) have a physics and characteristic scales that make their explicit modeling too computationally challenging or even unviable (see Chapter 10 of [27]). For this reason, parametrization of some sort is a common option in use within numerical models. Microphysical parametrization relies on the physical knowledge of certain processes without the need for a full computation of all the microscale processes that are involved. The clear limitation is that certain phenomena cannot be represented with high accuracy if they lay outside of the conditions required by the parametrization. The most common representation of cloud microphysics was designed by Kessler in [17].

Kessler's is a *bulk* model, meaning that water species are categorized only with respect to the particles type. It is a simple scheme based on the *warm rain* assumption for which ice is not considered. The main limitation of the warm condition is that only moist convection at the tropics, or mid-latitudes in the warm season, can be represented. The three forms of water considered here are (i) water vapor, (ii) cloud water that consists of water droplets whose size is so small that its terminal fall speed is negligible, and (iii) precipitating water that only includes rain (namely, drops whose diameter is > 0.5 mm). Drizzle (rain of drop diameter between 0.2 and 0.5 mm) is excluded.

The notation and the thermodynamics of water quantities is briefly introduced. The details of moist convection are found in [28,27,29]. In the absence of ice, water substances in the atmosphere are treated in terms of density of water vapor, ρ_v, density of cloud water, ρ_c, and density of rain, ρ_r. Given a mass per unit volume of dry air, ρ_d, the mixing ratios of vapor, cloud, and rain are

$$q_i = \frac{\rho_i}{\rho_d}, \quad i = v, c, r. \tag{1}$$

Pressure of moist air, p, is the sum of the partial pressure of dry air and the partial pressure of vapor. We have:

$$p = \rho_d R_d T\left(1 + \frac{q_v}{\varepsilon}\right), \tag{2}$$

where T indicates temperature, $R_d = 287\ \mathrm{J\,kg^{-1}\,K^{-1}}$ is the gas constant of dry air, $\varepsilon = R_d/R_v$, and $R_v = 461\ \mathrm{J\,kg^{-1}\,K^{-1}}$ is the gas constant of water vapor. Density temperature is defined as

$$T_\rho = T\frac{1 + q_v/\varepsilon}{1 + q_t}, \tag{3}$$

where $q_t = q_v + q_c + q_r$. From (3), density potential temperature is

$$\theta_\rho = T_\rho\left(\frac{p_0}{p}\right)^{R_d/c_{pd}}, \tag{4}$$

where $p_0 = 10^5$ Pa and $c_{pd} = 1004\ \mathrm{J\,kg^{-1}\,K^{-1}}$ is the specific heat of dry air at constant pressure. In terms of θ_ρ, the equation of state becomes

$$p = p_0\left(\frac{R_d\rho_d\theta_\rho}{p_0}\right)^{c_{pd}/c_{vd}}, \tag{5}$$

where $c_{vd} = 717\ \mathrm{J\,kg^{-1}\,K^{-1}}$ is the specific heat of dry air at constant volume.