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Block-spectral mapping for multi-scale solution

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ABSTRACT

In this paper, an efficient multi-scale method is presented. Discrete block domains with high mesh resolution are used to resolve fine scale features and the number of the blocks to be solved is kept as small as being sufficient for constructing a block–block spatial spectrum. A pointwise spectrum for the blockwise variation is generated for each mesh point on the corresponding block boundaries. These block-to-block spectra provide the required formation for all the block interfaces, and hence enable the fine scale solutions in these solved blocks to be mapped to a large domain simultaneously. The structured blocking also provides a convenient domain division for parallel processing. The method has been implemented in a 3-D time-marching finite volume solver for the Reynolds-averaged compressible Navier–Stokes equations. The double Fourier series is adopted to construct the pointwise spatial spectrum, which is well suited for wall bounded problems where a domain layer with fine scale resolution is required. Several test cases of mass and heat transfer problems have been examined and the results show consistently that the accurate solutions for fine scale domains can be obtained very efficiently with a reduction in degrees of freedom by two orders of magnitude, illustrating the potential of solving complex problems which might currently be intractable.

1. Introduction

There is a wide range of problems of practical interest, where the length scales to be dealt with are hugely disparate. In many cases, there are two distinctive and very different length scales. The fine scale features of a huge number of geometrically largely similar regions collectively interact with globally large scale phenomena. Examples include flows through porous medium, effusion film-cooling through many holes of a tiny size, acoustical liners for duct noise reduction, and surface mini-scale treatment for flow control (e.g., dimples etc.). The traditional treatment of this kind of problems would be to use empirically based models to count for the effects of the small scale features and to solve an up-scaled problem on a coarse mesh. The generality of the solutions following this kind of approaches are of course limited by the very empirical nature of these fine-scale models. In particular, when the behavior of the problem of interest is dependent on the detailed fine scale configurations, a detailed analysis for understanding of the causal link between the fine-scale structure and large scale behavior would be difficult to get. A further task of altering the fine-scale geometry configuration for performance sensitivity analysis and optimization would be even more difficult.

In recent years, there have been active developments in multi-scale methods [1–7]. Hou and Wu [1] proposed the multi-scale model based on a finite element formulation, in which a local fine mesh solution is used to generate the basis function linking to the corresponding coarse mesh variables, which is then used to construct the whole domain multi-scale solution based on the coarse mesh solution. The principal multi-scale approach has been shown to produce good results for problems

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Nomenclature

A_{mn}, B_{mn}, C_{mn}, D_{mn} coefficients of harmonics in the double Fourier series
A_T amplitude of inlet stagnation temperature temporal variation
M_B total number of fine scale blocks in the first index
M_b number of solved fine scale blocks in the first index
M_h number of Fourier harmonics in the first index.
N_B total number of fine scale blocks in the second index.
N_b number of solved fine scale blocks in the second index.
N_h number of Fourier harmonics in the second index.
P_{01} inlet stagnation pressure
T_{01} inlet stagnation temperature
t physical time
U conservative flow variables
τ pseudo time
ω angular frequency

Subscripts
0 stagnation parameters; time or space averaged
I, J block indices
i, j block surface mesh point indices
m, n Harmonics indices for the double Fourier series

with a huge scale disparity. Similar developments have also been made using the finite volume formulations by Jenny et al. [4,6,7]. An important property of the finite volume discretization scheme is that the conservation is maintained in the formulation. A further iterative process has been introduced to reduce the errors caused by the localization treatment of the fine scale solution for highly anisotropic problems [6]. The further iterations come with extra costs, and in a more recent development, the computational efficiency is increased by local solution-based adaptive iterations [7]. The previous developments in various forms, though largely based in incompressible flow models, have clearly indicated the potential of the multi-scale models in solving problems which would otherwise be intractable.

In the present work, a new multi-scale modeling technique has been developed in conjunction with a finite volume method. The main intended attribute of the model is that the same numerical discretization scheme and integration method are used for both the coarse and fine scales, so that the numerical resolution is consistently and completely dictated by the mesh scales. A blocking of the fine resolution domain is introduced to facilitate the two basic and somewhat competing requirements:

(a) accurate fine resolution of capturing fine scale flow features;
(b) avoidance of having to have fine mesh resolution for a large domain.

The way to gain the computational efficiency as the primary objective of all multi-scale modeling methods is now transformed to that to generate an efficient and accurate block-to-block spectrum to enable a mapping of the fine block solution to a large domain. The introduction of the fine mesh blocking structure gives an advantage that the model will fit in well with most existing multi-block flow solvers, only needing small modifications in updating the block boundary interfaces and some straightforward summations for generating the block-to-block spectrum. In a recent development, a block spectral model has been developed to address a more specific film-cooling problem, as implemented in a finite volume compressible flow solver in conjunction with a single-variable Fourier spectrum [8]. The accuracy and effectiveness of the technique demonstrated in this previous work indicates its potential for other multi-scale problems. In the present work, the method is extended to more general situations with a 2D (double) Fourier spatial spectrum and is examined and demonstrated for some mass and heat transfer test cases of general interest.

In the following sections, the modeling consideration for the block spectral approach is introduced first, followed by descriptions of the formulation, implementation and example results.

2. Modelling methodology

2.1. Two-scale problem

We consider the problems with a distinctive disparity in length scales and a similarity in geometrical and flow patterns for the small scale regime. For a non-uniform flow through a layer of porous medium the length scale of pores is much