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Moving boundary problems for a rarefied gas: Spatially one-dimensional case

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ABSTRACT

Unsteady flows of a rarefied gas in a full space caused by an oscillation of an infinitely wide plate in its normal direction are investigated numerically on the basis of the Bhatnagar–Gross–Krook (BGK) model of the Boltzmann equation. The paper aims at showing properties and difficulties inherent to moving boundary problems in kinetic theory of gases using a simple one-dimensional setting. More specifically, the following two problems are considered: (Problem I) the plate starts a forced harmonic oscillation (forced motion); (Problem II) the plate, which is subject to an external restoring force obeying Hooke's law, is displaced from its equilibrium position and released (free motion). The physical interest in Problem I lies in the propagation of nonlinear acoustic waves in a rarefied gas, whereas that in Problem II in the decay rate of the oscillation of the plate. An accurate numerical method, which is capable of describing singularities caused by the oscillating plate, is developed on the basis of the method of characteristics and is applied to the two problems mentioned above. As a result, the unsteady behavior of the solution, such as the propagation of discontinuities and some weaker singularities in the molecular velocity distribution function, are clarified. Some results are also compared with those based on the existing method.

1. Introduction

Moving boundary problems are one of the hot subjects in kinetic theory of gases and have been investigated extensively, in particular, in connection with micro electro mechanical systems (MEMS) [1]. The examples include the force on vibrating micro components exerted by the surrounding gas, the propagation of a sound wave generated by high-frequency oscillation of the boundary, the motion of vanes of the Crookes radiometer, etc. For moving boundary problems, which are essentially time-dependent, the prevailing direct simulation Monte Carlo (DSMC) method [2,3] is not an optimal method because one has to take the ensemble average over many independent runs in order to reduce the fluctuation inherent to the method (see, for instance, [4–6] for the application of DSMC method to moving boundary problems). Therefore, deterministic methods based on the model Boltzmann equations are usually employed with the help of known techniques in computational fluid dynamics (CFD), such as the moving mesh technique and the immersed boundary method [7–10].

In time-independent problems where the boundary is not moving in the normal direction, the molecular velocity distribution function on the plane or convex boundary (convex toward the domain of gas) is discontinuous at the molecular velocities tangent to the boundary (here, we are considering the case in which no external force acts on the gas molecules). For the convex boundary, this discontinuity propagates into the gas along the characteristics of the Boltzmann equation [11–13]. In contrast, for the plane boundary, the discontinuity stays on the boundary without propagating into the gas. The same is true

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when the plane boundary is moving in any direction with a constant velocity. However, if the boundary is accelerated in the direction opposite to the domain of the gas, the discontinuity is left in the gas even for the plane boundary, since the characteristics tangent to the boundary at a time go into the gas domain subsequently. If the boundary oscillates in the normal direction, therefore, the velocity distribution function may exhibit highly complex shape with many discontinuities as well as steep changes even for the plane boundary. To the best of the authors' knowledge, no attention has been paid to this point in the literature.

In the present study, we consider this problem. Restricting ourselves to spatially one-dimensional problems, we try to develop an accurate numerical method that is capable of describing the discontinuities in the molecular velocity distribution function generated by a moving plane boundary, on the basis of the Bhatnagar–Gross–Krook (BGK) model [14,15] of the Boltzmann equation. More specifically, we consider two problems: One is the unsteady gas motion in a half space produced by a forced harmonic oscillation of the plane wall in its normal direction, and the other is the decay of a one-dimensional oscillator (linear pendulum) caused by the drag force exerted by the gas. The former is nothing but the problem of nonlinear acoustic wave propagation [5,16]. In the latter problem, which is a sort of free-boundary (or coupled-boundary) problem and has been investigated extensively for a free-molecular (or Knudsen) gas [17,18], our final purpose is to find the correct decay rate of the amplitude of the oscillator.

The aim of the present paper is not to develop an efficient numerical scheme but to solve the above basic problems faithfully and establish reliable numerical solutions, at least for the BGK model, that may serve as reference solutions when efficient numerical methods are devised. Here, we should note that the BGK model has served as an important precursor for the study of the full Boltzmann equation. Once efficient numerical schemes for the evaluation of the collision integral of the Boltzmann equation are available, they may be implemented in place of the BGK collision term in the present approach.

The paper is organized as follows. We first formulate the two problems in Section 2. Section 3 is devoted to some preliminary discussions for numerical analysis. We develop the numerical method in Section 4, and summarize the results of numerical analysis in Section 5. Some concluding remarks are given in Section 6.

2. Formulation of the problem

2.1. Problem, assumptions, and notations

Let us consider an infinitely wide plate without thickness, kept at temperature T_{0*} and immersed in an infinite expanse of a rarefied ideal monatomic gas in an equilibrium state at rest with temperature T_{0*} and density ρ_{0*}. We take the X_1 axis of the Cartesian coordinate system X_i perpendicular to the plate. At time $t_* = 0$, where t_* is the time variable, the plate is set into motion in the X_1 direction in a manner described below. We investigate the subsequent motion of the gas numerically under the following assumptions:

(i) The behavior of the gas is described by the BGK model [14,15] of the Boltzmann equation.

(ii) The gas molecules undergo diffuse reflection on the plate. More specifically, the velocity of the reflected molecules on the boundary are distributed according to the (half-range) Maxwellian distribution being characterized by the velocity and temperature of the plate and having the density determined in such a way that there is no instantaneous net mass flow across the plate (see, e.g., [12,13]).

Let us denote by $X_w(t_*)$ the position (X_1 coordinate) of the plate and by $V_w(t_*)$ its velocity, i.e., $V_w(t_*) = \dot{X}_w(t_*)$, where a dot indicates the time derivative. In the present study, we consider the following two types of motion of the plate:

[Problem I] (forced motion): The plate starts a forced harmonic oscillation given by

$$X_w(t_*) = a_* \cos \omega_* t_*, \tag{1}$$

at $t_* = 0$ (see Fig. 1(a)), where a_* and ω_* are the amplitude and angular frequency of the oscillation. In this problem, we practically consider the half space $X_1 \geqslant X_w(t_*)$.

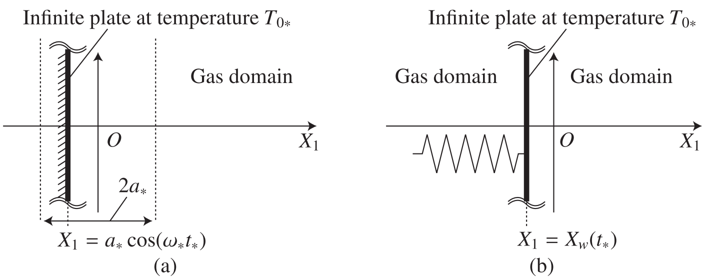

Fig. 1. Configuration of the problem: (a) forced motion (Problem I), (b) free motion (Problem II).