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An adaptive edge finite element method for electromagnetic cloaking simulation ☆

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ABSTRACT

In this paper we develop an adaptive edge finite element method based on a reliable and efficient recovery type a posteriori error estimator for time-harmonic Maxwell equations. The asymptotically exact a posteriori error estimator is based on the superconvergence result proved for the lowest-order edge element on triangular grids, where most pairs of triangles sharing a common edge form approximate parallelograms. The efficiency and robustness of the proposed method is demonstrated by extensive numerical experiments for electromagnetic cloaking problems with highly anisotropic permittivity and permeability.

1. Introduction

In early 2006, Pendry et al. [33] and Leonhardt [24] independently presented the blueprints for making objects invisible to electromagnetic waves by using metamaterials. The basic idea is to use the Maxwell equations' form invariant property to design the permittivty and permeability of the metamaterial. The cloaked region is surrounded by this cloaking metamaterial, and the light will be guided around the cloaked region as if nothing were there. In late 2006, the first practical realization of such a cloak was demonstrated by Schurig et al. [35] over a band of microwave frequencies for a 2-D cloak constructed using artificially structured metamaterials. At present, there is a tremendous interest in the study of invisibility cloaks and their striking applications. We refer readers to (e.g., [8,15,16,29,23,28,30]) for more details about this rapidly growing field and more complete literature.

Numerical simulation plays a very important role in designing the invisibility cloaks and validating theoretical predictions. Due to its flexibility in handling complex geometrical domains and its solid mathematical theory, the finite element method (FEM) is one of the most popular techniques in solving electromagnetic wave propagation problems. To date, the FEM cloaking simulation seems to be dominated by the commercial package COMSOL. To our best knowledge, not much research has been devoted to developing more efficient and robust FEMs for cloaking simulation. In 2011, Zhai et al. [40] developed an efficient finite element method for solving 3D axisymmetrical invisibility cloaks and concentrators. Recently, Demkowicz and Li [12] extended the Discontinuous Petrov–Galerkin (DPG) method to show the effectiveness of the DPG method in cloak simulations. Some simple h-adaptivity has been introduced for our DPG method, however theoretical justification of the effectiveness of h-adaptive was not investigated there.

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In this paper, a reliable and efficient recovery type a posteriori error estimator is developed for time-harmonic Maxwell's equations. Our estimator is the so-called recovery type, which was originally introduced by Zienkiewicz and Zhu [42] and has been extensively developed and analyzed by various authors [38,39] for many partial differential equations except Maxwell's equations. More specifically, our error estimator is based on the superconvergence result obtained for time-harmonic Maxwell's equations solved by the lowest order triangular edge element. Comparing to those existing superconvergence results for Maxwell's equations (e.g., [17,34], [32, p. 201], and [[27, Ch. 5]), our superconvergence result is original. We then use this a posteriori error estimator in our adaptive edge element method and the effectiveness is demonstrated by several cloaking simulations. We like to remark that there are some excellent works on a posteriori error estimators [4,7,9,10,41] and adaptive FEM [1,11,21,36] for Maxwell's equations, but they are mainly the residual types for a simple medium such as vacuum and have not used for such complicated problems as our cloaking simulations.

The rest of the paper is organized as follows. In Section 2, we provide the superconvergence analysis for time-harmonic Maxwell's equations solved by the lowest order triangular edge element. Then in Section 3, we apply the superconvergence result to derive the recovery type a posteriori error estimator, and use the error estimator to develop an adaptive finite element method for electromagnetic cloaking simulations. Extensive numerical experiments are presented in Section 4 to justify our theoretical analysis and demonstrate the effectiveness of our adaptive method. We conclude the paper in Section 5.

2. Superconvergence analysis for time-harmonic Maxwell's equations

First let us introduce some common notations. We assume that Ω is a bounded and simply connected Lipschitz polyhedron of R^d ($d = 2$ or 3) with connected boundary $\partial\Omega$ and unit outward normal $\boldsymbol{n}$. For $m, p \geqslant 1$, we denote the standard Sobolev space by $W^{m,p}(\Omega)$. When $p = 2$, we usually write $H^m(\Omega) = W^{m,2}(\Omega)$. Furthermore, we need some other Sobolev spaces:

$$H_0(curl;\Omega) = \{\boldsymbol{v} \in (L^2(\Omega))^d; \quad \nabla\times\boldsymbol{v} \in (L^2(\Omega))^d,\ \boldsymbol{n}\times\boldsymbol{v} = \boldsymbol{0} \text{ on } \partial\Omega\},$$
$$H^s(curl;\Omega) = \{\boldsymbol{v} \in (H^s(\Omega))^d; \quad \nabla\times\boldsymbol{v} \in (H^s(\Omega))^d\},\ \forall s > 0.$$

The above spaces are equipped with norms

$$||\boldsymbol{v}||_{H(curl;\Omega)} = (||\boldsymbol{v}||_0^2 + ||\nabla\times\boldsymbol{v}||_0^2)^{1/2} \quad \forall\boldsymbol{v} \in H_0(curl;\Omega),$$
$$||\boldsymbol{v}||_{H^s(curl;\Omega)} = (||\boldsymbol{v}||_{H^s(\Omega)}^2 + ||\nabla\times\boldsymbol{v}||_{H^s(\Omega)}^2)^{1/2} \quad \forall\boldsymbol{v} \in H^s(curl;\Omega),$$

where $||\cdot||_{0,\Omega}$ (or simply $||\cdot||_0$) denotes the $(L^2(\Omega))^d$ norm.

2.1. The modeling equations and some preliminaries

Modeling of electromagnetic phenomena at a fixed frequency ω is governed by the full Maxwell's equations:

$$\nabla\times\boldsymbol{E} + i\omega\mu\boldsymbol{H} = \boldsymbol{0}, \quad \text{in } \Omega, \tag{2.1}$$
$$\nabla\times\boldsymbol{H} - i\omega\varepsilon\boldsymbol{E} = \boldsymbol{J}, \quad \text{in } \Omega, \tag{2.2}$$

where $i = \sqrt{-1}$, $\boldsymbol{E}(\boldsymbol{x})$ and $\boldsymbol{H}(\boldsymbol{x})$ are the electric and magnetic fields, ε and μ are the permittivity and permeability of the material, and $\boldsymbol{J}$ is the applied current density.

Eliminating $\boldsymbol{H}$ from (2.1) and (2.2), we obtain

$$\nabla\times(\mu^{-1}\nabla\times\boldsymbol{E}) - \omega^2\varepsilon\boldsymbol{E} = -i\omega\boldsymbol{J} \quad \text{in } \Omega. \tag{2.3}$$

Let us denote the wavenumber $k = \omega\sqrt{\varepsilon\mu}$. To avoid the technicality and simplify the presentation, we assume that ϵ and μ are constants, in which case we can simplify the problem (2.3) to:

$$\nabla\times(\nabla\times\boldsymbol{E}) - k^2\boldsymbol{E} = \boldsymbol{F} \equiv -i\omega\mu\boldsymbol{J} \quad \text{in } \Omega. \tag{2.4}$$

Moreover, we assume that the problem (2.4) is subject to the perfectly conducting (PEC) boundary condition

$$\boldsymbol{n}\times\boldsymbol{E} = \boldsymbol{0} \quad \text{on } \partial\Omega. \tag{2.5}$$

The variational formulation of the problem (2.4) and (2.5) is to find $\boldsymbol{E} \in H_0(curl;\Omega)$ such that

$$a(\boldsymbol{E},\boldsymbol{\phi}) = (\boldsymbol{F},\boldsymbol{\phi}) \quad \forall\boldsymbol{\phi} \in H_0(curl;\Omega), \tag{2.6}$$

where

$$a(\boldsymbol{E},\boldsymbol{\phi}) = (\nabla\times\boldsymbol{E},\nabla\times\boldsymbol{\phi}) - k^2(\boldsymbol{E},\boldsymbol{\phi}). \tag{2.7}$$

Here and below $(\cdot,\cdot)$ denotes the inner product in $(L^2(\Omega))^d$.

To design a finite element method, we assume that Ω is partitioned by a regular mesh T_h of tetrahedra in R^3 (or triangles in R^2), where h is the mesh size. Due to the low regularity of the solution for Maxwell's equations, we just consider the lowest-order Nédélec curl conforming element (often called edge element) space:

$$\boldsymbol{V}_h = \{\boldsymbol{\phi}_h \in H_0(curl;\Omega) : \quad \boldsymbol{\phi}_h|_K = \text{span}\{\lambda_i\nabla\lambda_j - \lambda_j\nabla\lambda_i\},\ \forall K \in T_h\}, \tag{2.8}$$