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A discontinuous Galerkin conservative level set scheme for interface capturing in multiphase flows

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ABSTRACT

The accurate conservative level set (ACLS) method of Desjardins et al. [O. Desjardins, V. Moureau, H. Pitsch, An accurate conservative level set/ghost fluid method for simulating turbulent atomization, J. Comput. Phys. 227 (18) (2008) 8395–8416] is extended by using a discontinuous Galerkin (DG) discretization. DG allows for the scheme to have an arbitrarily high order of accuracy with the smallest possible computational stencil resulting in an accurate method with good parallel scaling. This work includes a DG implementation of the level set transport equation, which moves the level set with the flow field velocity, and a DG implementation of the reinitialization equation, which is used to maintain the shape of the level set profile to promote good mass conservation. A near second order converging interface curvature is obtained by following a height function methodology (common amongst volume of fluid schemes) in the context of the conservative level set. Various numerical experiments are conducted to test the properties of the method and show excellent results, even on coarse meshes. The tests include Zalesak's disk, two-dimensional deformation of a circle, time evolution of a standing wave, and a study of the Kelvin–Helmholtz instability. Finally, this novel methodology is employed to simulate the break-up of a turbulent liquid jet.

1. Introduction

In simulations of multiphase flows, changing fluid properties and surface tension at the interface lead to discontinuities that make discretizing the Navier–Stokes equations challenging. Consequently, numerical methods have been developed to handle these singularities, including the continuum surface force (CSF) approach [1] and the ghost fluid method (GFM) [2]. Both the CSF method and the GFM are based on the assumption that the interface location is known accurately. The discontinuous Galerkin conservative level set method, presented herein, provides an accurate interface location needed for the CSF method, the GFM, or other chosen methods.

Commonly, two classes of methods are used to locate the interface: interface tracking and interface capturing. Interface tracking schemes typically use either arbitrary Lagrangian–Eulerian (ALE) methods based on a mesh that deforms with the interface [3–6] or marker and cell (MAC) methods that advect Lagrangian particles that define a given fluid by their locations [7]. The main problem with interface tracking schemes occurs when the interface deforms substantially or when the interface disconnects and reconnects. Significant re-meshing or re-seeding of particles is needed to account for the large interface changes.

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Interface capturing methods include volume of fluid (VOF) and level set methods. VOF methods capture the liquid volume fraction in each grid cell [8–10]. While VOF schemes have excellent mass conservation properties, they suffer from the challenge of reconstructing the interface location using only the cell volume fraction. Level set methods represent the interface as an iso-surface of a function called the level set function [11,12]. Level set methods alleviate the problem found with VOF methods of having to reconstruct an interface since the interface location, normal, and curvature are readily accessible from the level set function. Classically, the level set function is defined as a signed distance function, which lacks conservation properties. The conservation properties were improved with the development of the conservative level set [13–15]. Details of the signed distance and conservative level set methods are given in Section 2.

Spatial discretization of the conservative level set can be performed using finite difference operators, which was done in the accurate conservative level set (ACLS) method [15]; however, the discontinuous Galerkin (DG) discretization method was chosen in this work for its high accuracy and compact stencil [16,17]. High accuracy is obtained by projecting the solution onto high-order discontinuous polynomials, similar to finite element methods. Compactness is a result of the local nature of the polynomials. Since the polynomials are defined on each grid cell, updates do not need global information but rather only information from nearest grid cell neighbors. This small stencil results in minimal communication requirements and a highly parallelizable code. The small stencil also makes the discretization amenable to both structured and unstructured grids. Although a Cartesian structured grid is the focus of this work, the extension of the proposed framework to an unstructured grid is possible. The aforementioned scheme that discretizes the conservative level set using a DG formulation is refered to as the discontinuous Galerkin conservative level set (DG-CLS).

The conservative level set method includes a transport equation that describes the convection of the level set due to the fluid velocity and a reinitialization equation that maintains the shape of the level set. Cockburn and Shu [18] provide a DG discretization of the transport equation with an accurate temporal integration method and appropriate definition of fluxes. The DG discretization was applied to the signed distance level set by Marchandise et al. [19]. A quadrature-free implementation was used wherein all the integrals that appear in the weak form of the equations were precomputed to improve computational efficiency. Marchandise et al. [20] reinitialized the signed distance level set using a recursive contouring algorithm with a fast search tree method to find the smallest distance to the interface, which, for the signed distance level set method, is also the value of the level set function. However, when the conservative level set is used the level set is not a signed distance function and a different reinitialization method is employed. Following the steps of the ACLS method, a compressive-diffusion equation is solved to reinitialize the level set and maintain its profile. We propose to discretize the compressive-diffusion equation using DG in order to maintain a similar order of accuracy for both transport and reinitialization steps. Details of the DG implementation are given in Section 3 which includes background information on our DG formulation in Section 3.1 and the particulars of the spatial discretization of the transport and reinitialization equations in Sections 3.2 and 3.3, respectively.

In Section 4, the stability of the DG-CLS scheme is discussed. We begin the section with a discussion of how to improve the robustness of the scheme when the discretized level set function oscillates above or below the bounds due to the use of high order polynomials. Then, an investigation on the effect of a minimum/maximum preserving limiter [21] is provided. The limiter is designed to improve the boundedness of the conservative level set function. While the limiter is found to improve the boundedness, it significantly reduces the accuracy of the method.

The most straightforward method to integrate the transport and reinitialization equations in time is an explicit scheme such as a Runge–Kutta (RK) method. An explicit scheme does not require global communications, thereby maintains the highly parallelizable nature of the DG spatial discretization. Cockburn and Shu [18] provide a description of the RK methods and show many are stable when high order polynomials are used in the DG approximation of the solution. The total variation diminishing third order RK (TVD-RK3) method is used in this work; details are provided in Section 5.

The interface curvature and normal have direct effects on the solution; therefore, the methods used to calculate these interface properties should be accurate and converge under mesh refinement. In the context of their DG implementation of the signed distance level set method, Marchandise et al. [20] proposed a least squares approach for computing curvature. This method was later employed by Desjardins et al. [15] in the ACLS method, but only first order curvature convergence was observed. Obtaining convergence is difficult with the conservative level set because the level set profile is a relatively sharp approximation of a step function. In fact, sharper profiles lead to smaller conservation errors [15]. When the mesh is refined, one has to choose between sharpening the level set profile such that the number of cells across the profile remains constant, or keeping the profile unchanged thereby improving the resolution of the profile. Choosing the former leads to improved mass conservation, while the latter leads to lower curvature errors. To improve the convergence of the interface curvature while sharpening the level set profile, we applied the height function method commonly used in volume of fluid (VOF) methods [22] to the DG-CLS formulation resulting in a near second order converging curvature. Details of the implementation are described in Section 7.

In Sections 6 and 9, we discuss numerical experiments conducted using the DG-CLS method. Section 6 includes tests that focus on the DG-CLS method and Section 9 has applications that use the DG-CLS scheme coupled with the NGA flow solver [23]. Coupling between DG-CLS and the flow solver is presented in Section 8. The numerical tests include normal and curvature convergence, simulations that examine transport of the level set function, and several canonical multiphase flow problems.