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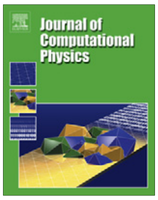

Fluid–structure interaction in blood flow capturing non-zero longitudinal structure displacement

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ABSTRACT

We present a new model and a novel loosely coupled partitioned numerical scheme modeling fluid–structure interaction (FSI) in blood flow allowing non-zero longitudinal displacement. Arterial walls are modeled by a linearly viscoelastic, cylindrical Koiter shell model capturing both radial and longitudinal displacement. Fluid flow is modeled by the Navier–Stokes equations for an incompressible, viscous fluid. The two are fully coupled via kinematic and dynamic coupling conditions. Our numerical scheme is based on a new modified Lie operator splitting that decouples the fluid and structure sub-problems in a way that leads to a loosely coupled scheme which is unconditionally stable. This was achieved by a clever use of the kinematic coupling condition at the fluid and structure sub-problems, leading to an implicit coupling between the fluid and structure velocities. The proposed scheme is a modification of the recently introduced “kinematically coupled scheme” for which the newly proposed modified Lie splitting significantly increases the accuracy. The performance and accuracy of the scheme were studied on a couple of instructive examples including a comparison with a monolithic scheme. It was shown that the accuracy of our scheme was comparable to that of the monolithic scheme, while our scheme retains all the main advantages of partitioned schemes, such as modularity, simple implementation, and low computational costs.

1. Introduction

We study fluid–structure interaction (FSI) between an incompressible viscous, Newtonian fluid, and a thin viscoelastic structure modeled by the linearly viscoelastic cylindrical Koiter shell model. The cylindrical viscoelastic Koiter shell model is derived to describe the mechanical properties of arterial walls, while the Navier–Stokes equations for an incompressible, viscous, Newtonian fluid were employed to model the flow of blood in medium-to-large human arteries. The two are coupled via the kinematic (no-slip) and dynamic (balance of contact forces) coupling conditions. Motivated by recent results of *in vivo* measurements of arterial wall motion [1–4], which indicate that both the radial and longitudinal displacement, as well as viscoelasticity of arterial walls, are important in disease formation, we derived in this work the viscoelastic cylindrical Koiter shell model which captures both radial and longitudinal displacement, with the viscoelasticity of Kelvin–Voigt type. The novel Koiter shell model is then coupled to the Navier–Stokes equations, and the coupled FSI problem is solved numerically. In this manuscript we devise a stable, loosely coupled scheme to numerically solve the fully coupled FSI problem. The scheme is

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based on a novel modified Lie's time-splitting, and on an implicit use of the kinematic coupling condition, as in [5], which provides stability of the scheme without the need for sub-iterations between the fluid and structure sub-solvers. Stability of the scheme was proved in [6] on the same, simplified benchmark problem as in [7]. We provide numerical results which show that the scheme is first-order accurate in time. We compare our results with the monolithic scheme of Quaini [8] and Badia et al. [9] showing excellent agreement and comparable accuracy.

In hemodynamics, the coupling between fluid and structure is highly nonlinear due to the fact that the fluid and structure densities are roughly the same, making the inertia of the fluid and structure roughly equal. In this regime, classical partitioned loosely coupled (or explicit) numerical schemes, which are based on the fluid and structure sub-solvers, have been shown to be intrinsically unstable [7] due to the miss-match between the discrete energy dictated by the numerical scheme, and the continuous energy of the coupled problem. This has been associated with the explicit role of the "added mass effect", introduced and studied in [7]. To rectify this problem, the fluid and structure sub-solvers need to be iterated until the energy balance at the discrete level approximates well the energy of the continuous coupled problem. The resulting strongly coupled partitioned scheme, however, gives rise to extremely high computational costs.

To get around these difficulties, several different loosely coupled algorithms have been proposed that modify the classical strategy in coupling the fluid and structure sub-solvers. The method proposed in [10] uses a simple membrane model for the structure that can be easily embedded into the fluid equations and appears as a generalized Robin boundary condition. In this way the original problem reduces to a sequence of fluid problems with a generalized Robin boundary condition that can be solved using only the fluid solver. A similar approach was proposed in [11], where the fluid and structure are split in the classical way, but the fluid and structure sub-problems were linked via novel transmission (coupling) conditions that improve the convergence rate. Namely, a linear combination of the dynamic and kinematic interface conditions was used to artificially redistribute the fluid stress on the interface, thereby avoiding the difficulty associated with the added mass effect.

A different stabilization of the loosely coupled (explicit) schemes was proposed in [12] which is based on Nitsche's method [13] with a time penalty term giving L^2-control on the fluid force variations at the interface. We further mention the scheme proposed in [14], where Robin-Robin type preconditioner is combined with Krylov iterations for the solution of the interface system.

For completeness, we also mention several semi-implicit schemes. The schemes proposed in [15–17] separate the computation of fluid velocity from the coupled pressure-structure velocity system, thereby reducing the computational costs. Similar schemes, derived from algebraic splitting, were proposed in [9,18]. We also mention [19] where an optimization problem is solved at each time-step to achieve continuity of stresses and continuity of velocity at the interface.

In our work we deal with the problems associated with the added mass effect by: (1) employing the kinematic coupling condition implicitly in all the sub-steps of the splitting, as in the kinematically coupled scheme first introduced in [5]; (2) treating the fluid sub-problem together with the viscous part of the structure equations so that the structure inertia appears in the fluid sub-problem (made possible by the kinematic coupling condition), giving rise to the energy estimates that mimic those in the continuous problem. In this step, a portion of the fluid stress and the viscous part of the structure equations are coupled weakly, and implicitly, thereby adding dissipative effects to the fluid solver and contributing to the overall stability of the scheme (although the scheme is stable even if viscoelasticity of the structure is neglected). The modification of the Lie splitting introduced in this manuscript uses the remaining portion of the normal fluid stress (the pressure) to explicitly load the structure in the elastodynamics equations, significantly increasing the accuracy of our scheme when compared with the classical kinematically coupled scheme [5], and making it comparable to that of the monolithic scheme presented in [8,9].

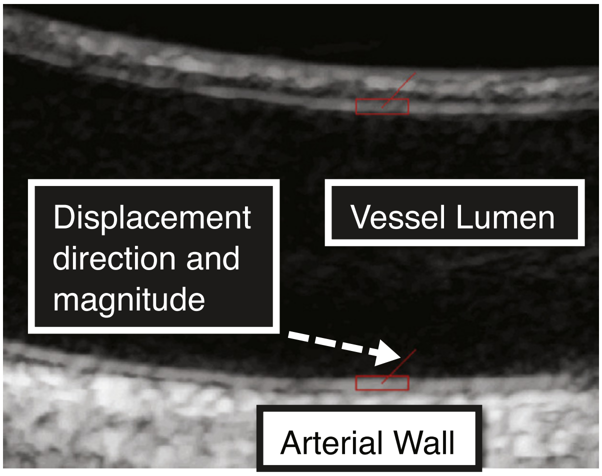

Fig. 1. Longitudinal displacement in a carotid artery measured using *in vivo* ultrasound speckle tracking method. The thin red line located at the intimal layer of the arterial wall shows the direction and magnitude of the displacement vector, showing equal magnitude in longitudinal and radial components of the displacement [20].