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The diamond of contraries ☆

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ABSTRACT

A number of approaches for logical reasoning with diagrams have been proposed. This paper considers the question, how the expressiveness of such systems can be raised without losing the visual power of less expressive diagrams. The antagonism between expressiveness and diagrammatic simplicity is coped with by a set of jointly exhaustive and contrary relations, modelling definite knowledge within a new diagrammatic representation. The restriction on actual knowledge reduces the expressiveness of these diagrams, but strengthens their visual power by avoiding ambiguities and by providing a close correspondence between diagrammatic syntax and set-theoretic semantics. The extension towards compound diagrams enables the representation of uncertain knowledge, but have a negative impact on the clarity of these diagrams. It is shown how formulae of monadic first-order logic are treated within this approach.

1. Introduction

Diagrammatic reasoning in logic has become an established area of research at the very latest since Shin's formalisation of Venn diagrams in the 1990s [24]. Diagrams in this field are predominantly region based [28,8,31,21,29,9,4], while less research has been carried out on linear diagrams [10,13,20,22,12] which offer an interesting alternative since they have other advantages [5,10]. This paper aims to put forward diagrammatic systems of the latter type.

A focus lies on the question, how the visual power of diagrams can be maintained when restricting the knowledge to be dealt with to actual and complete knowledge. This is in contrast to most conventional approaches like the Aristotelian logic which interpret the universal affirmative *all* A *is* B as an improper part relation among sets, implying both alternatives that A and B are equal and that B covers A. For the particular affirmative *some* A *is* B even four interpretations exist, letting the semantics of both affirmative statements overlap. All possible interpretations are explicitly told apart in a new diagrammatic representation in order to develop a logic that is inspired by Brendan Larvor who analyses the ambivalence of the particular affirmative [14]. In doing so, the chosen representation is confined to represent actual knowledge. This restricts the expressiveness of these diagrams, but strengthens their visual power by avoiding ambiguities of improper set relations and by providing a close correspondence between diagrammatic syntax and set-theoretic semantics through the existential import to these diagrams. The extension towards compound diagrams enables the representation of uncertain knowledge and increases the expressiveness towards first-order monadic logic.

1.1. Conjunctive information in diagrams

Besides the antagonism between the simplicity of diagrams and their expressiveness, there is a more specific trade-off between two desired features of diagrammatic systems. The first one concerns the possibility to integrate information in single diagrams (which relates to the expressiveness of diagrams), while the second one concerns the visual power of diagrams (which relates to the simplicity of diagrams). A conflict arises because

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established systems enable the integration of information in single diagrams by means of specific syntactical devices, however the more the syntactical devices there are the stronger the negative effect on the visual power of those diagrams. At least for the present purposes it seems reasonable to assume that the visual power decreases with the number of syntactical elements to be deployed.

Venn-I and Venn-II systems [24] as well as spider diagrams [4] allow the integration of several pieces of information in a diagram by specific syntactical devices that mark subsets as to be empty or non-empty or to represent disjunctive information. Conjunctive information can be represented within these diagrams, since several marks can be simultaneously employed for a single diagram. In detail, the following syntactical objects are employed:

- The emptiness of a set is represented by shading the corresponding region which represents that set. This results into bicolour diagrams with some areas being completely coloured.
- In order to assert that at least one element exists in a set, the corresponding region is marked by a specific symbol, such as an $\otimes$.
- Disjunctive information is represented by means of lines that connect $\otimes$'s which are found in different regions.
- Some systems enclose diagrams with rectangles to represent the background set.
- Venn diagrams require the consideration of all possible set intersections.
- Labels are depicted near the boundaries of the sets they denote.

Taking all objects required in order to describe a specific formula, the syntactical structure of such diagrams becomes quite complex and unclear. Ambiguities arise for the labels which are employed to denote the present regions, which are all near each other. Ambiguities also arise due to the overlap of different curves that enclose the regions. The more $\otimes$'s there are, the more difficult it is to oversee the information being represented. Already a simple diagram with only two basic regions and a couple of marks prevents one to derive anything effortlessly (p. 23 in [24]). The shading of entire areas is unpractical for handmade diagrams.

Although less expressive, Euler circles [8] have another advantage: the existential import to those diagrams provides a direct relationship between topological relations at the syntactical level and set-theoretic relations at the semantic level, making their comprehension very simple. Diagrams with this characteristic have been referred to as diagrams of the Euler-type as opposed to those of a Venn-type [19]. Euler's original approach, however, has the disadvantage that uncertain information cannot be represented and that the integration of conjunctive information in single diagrams is very limited.

This paper puts forward a diagrammatic system that allows for the depiction of conjunctive information in single diagrams, however without the introduction of additional syntactic devices. Simultaneously, the visual power of Euler-type diagrams is maintained for this system and can even be improved by omitting the explicit representation of improper part and improper overlap relations, avoiding ambiguities of Euler circles [24]. But this is at the expense of the expressiveness which lies in-between that one of the former types of region based diagrams and Euler circles. The expressiveness, however, can be regained by allowing for groups of diagrams in order to represent disjunctive information.

1.2. Linear diagrams

The presented system pertains to the class of linear diagrams which have been developed as an alternative to region based diagrams for the representation of sets. There are basically four types of linear diagrams when distinguishing the way how strong the graphical objects are restricted, which represent sets and the embedding space: curved lines in the two-dimensional plane [1], straight lines in the two-dimensional plane [7], parallel, straight lines in a two-dimensional unbounded plane [13], and parallel, straight lines in a two-dimensional bounded plane [20]. Such linear representations have their roots in the work of Leibniz [15,17], who favoured linear diagrams over intersecting circles [27].

Recently, *set space diagrams* have been introduced as a visual language to represent sets and their relations [10], motivated by Cheng's *probability space diagrams* from which their name derive [6]. Set space diagrams resemble the linear system already proposed by Lambert in 1764 [13]. But a fundamental distinction is that, unlike in Lambert's approach, each set space diagram represents a background set, among others to explicitly show the complement of sets. Universal statements are represented in Lambert's system similar as in set space diagrams by aligned and disjoint segments for the representation of universal affirmatives and universal negatives, respectively. However, existential statements are represented by dotted segments in Lambert's system, while dotted segments would be ambiguous for set space diagrams which allow segments to be disconnected. Conversely, the avoidance of disconnected segments would limit the representation of all possible relations among sets. Overcoming several difficulties of Lambert's system in this way and motivated by a plain layout that avoids overlapping objects which would impair their appearance, set space diagrams have revitalised linear diagrams. As all segments, which represent the sets, are to be arranged in parallel, their construction is quite direct and their inspection does not suffer from clutter due to overlapping objects.

1.3. Objectives

The aim of the current paper is to apply set space diagrams to logic reasoning and to analyse them when they are confined to actual and complete knowledge. This is a substantial restriction regarding their expressiveness, but a restriction which will be revealed later on by relocating the treatment of uncertain knowledge to the consideration of groups of diagrams. Such groups represent disjunctive information, while single diagrams keep neat and retain their visual power. The resulting diagrammatic system instantiates the idea of a logic represented by the *diamond of contraries* which avoids ambiguities of conventional systems. This is made possible by defining a basis of certain knowledge,