Sustainable Energy, Grids and Networks 13 (2018) 148–156

Contents lists available at ScienceDirect

Sustainable Energy, Grids and Networks

journal homepage: www.elsevier.com/locate/segan

Density forecasting of daily electricity demand with ARMA-GARCH, CAViaR, and CARE econometric models

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HIGHLIGHTS

- Probabilistic day-ahead forecasting of low voltage phase currents is analyzed.
- Mean-variance models are preferable to quantile and expectile regression models.
- The use of an ARMA-GARCH econometric model with exogenous inputs is proposed.
- A fast iterative LS estimation scheme is proposed that achieves the lowest errors.

ARTICLE INFO

Article history:
Received 28 August 2016
Received in revised form 27 November 2017
Accepted 24 January 2018
Available online 9 February 2018

Keywords:
Conditional mean–variance models
Density forecasting
Expectile regression
Quantile regression
Short-term load forecasting

ABSTRACT

The emerging need for risk-aware operational decisions on power systems calls for the development of accurate probabilistic load forecasting methods. To serve this purpose, various celebrated modeling approaches are applied from the field of economics where uncertainty forecasting has been a longstanding fundamental area of research. In particular, this paper proposes the use of ARMA-GARCH conditional mean–variance model in day-ahead forecasting and evaluates the CAViaR quantile regression model and the CARE expectile regression model as alternatives, with all of them incorporating exogenous inputs. In addition to the conventional quasi-maximum likelihood estimation (QMLE) of the ARMA-GARCH model, a special emphasis is put on least-squares (LS) based iterative and nonlinear estimation schemes. Empirical results are generated based on low-voltage side currents collected from transformers in the Netherlands, with the forecasts being assessed probabilistically via the continuous ranked probability score. Performance comparisons demonstrated improved results with the ARMA-GARCH model in relation to the others. Moreover, its estimation by means of the proposed iterative LS estimation method achieved the best forecast performance in a short runtime, thereby proven to be attractive for practical deployment.

1. Introduction

Probabilistic forecasting has lately been gaining increased attention in the scope of electricity demand prediction. The primary contributing factor to this recognition is the progressive electrification of energy use. In terms of energy consumption, electric vehicles pose the foremost challenge to electricity grids by consuming, in just a few hours, a comparable amount of energy to a household's daily requirement [1]. Concerning generation, integration of renewable energy sources into the grid gives rise to an intermittent production that consequently hampers predictability [2]. Due to this ceaseless boost in both supply and demand of electricity, it is becoming a necessity to adapt the current decision-making strategies, such as those on unit commitment, dispatch planning, load control, maintenance scheduling, and electricity pricing, to cope with the growing uncertainty in forecasts. Therefore, as opposed to a point forecast (e.g., the conditional mean) used in the conventional deterministic planning, the full probability distribution of the future demand is a crucial prerequisite for risk-aware planning.

With the vast majority of the forecasting literature still being limited to point forecasts, probabilistic (density) forecasting [3], at first, was predominantly researched in the fields of economics [4] and meteorology [5] where processes tend to exhibit considerable volatility. Owing to the outlined transition in the energy sector, it is gradually finding more place in electricity forecasting [2,6–16]. In particular, [7] provides a comprehensive review exclusively on this topic. Nevertheless, economics is arguably the leading field as far as discoveries in uncertainty forecasting are concerned and this

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https://doi.org/10.1016/j.segan.2018.01.001
2352-4677/

constitutes the basis for our motivation to investigate various well-established econometric models in the context of load forecasting.

The focus of density forecasting in this work is on day-ahead forecasts at a local (neighborhood) level, which falls under the category of short-term load forecasting in terms of the forecast horizon. Regarding the level of aggregation, phase currents from the low voltage cables of medium-to-low voltage transformers are of interest. The measurements are collected at every quarter of an hour from transformers in the Netherlands.

In the modeling process to produce the forecasts, a separate model is built for the evolution of each current. An outline of the problem context and the decision mechanism in modeling that will be explained from here onwards is provided in Fig. 1. Unlike in long-term forecasting, a modeling set ranging from several weeks to a few months in duration was determined to be the most relevant for the considered day-ahead forecasting. In this rather short set, nonlinearities among variables, largely as a result of differing seasonal correlations between weather variables and loads (e.g., in [17] and [18]), have not been detected. Therefore, as also indicated in our prior work [19], we limit our scope to linear strategies, and in particular, to *autoregressive moving average* (ARMA) [20] models due to having complications with exponential smoothing models when density forecasts [21] and exogenous inputs [22] are involved. If strong nonlinearities were present due to high impact of renewable sources, then computationally more intensive machine learning approaches (e.g., in [10,11]) would have been preferred. Although the ARMA structure with exogenous inputs (ARMAX) accounts for temporally correlated uncertainty, its homoscedasticity (constant variance conditional on the past data) assumption becomes invalid for many time-series, including ours. As the traditional route, heteroscedasticity is alleviated by transforming the data accordingly, such as with the logarithmic, square root and, most frequently, Box–Cox [23] transformations. However, the substantial diversity in demand patterns and the emphasis placed on volatility in density forecasting entails a richer, unified means of uncertainty quantification.

Along this line, we combine the ARMAX conditional mean model with the renowned *generalized autoregressive conditional heteroskedasticity* (GARCH) conditional variance model [24] from economics. Based on the conclusions in [19], the GARCH side of the model is also enriched by allowing to admit exogenous inputs, such as indicator variables to model hour-dependent variance levels. Furthermore, the analysis in [19] primarily concentrated on quasi-maximum likelihood estimation (QMLE) techniques where an additional estimation of the non-normality of innovations did not contribute to prediction performance. Therefore, we proceed with the standard normality-based QMLE and propose simpler estimation routines that employ least-squares (LS) optimizations as possible alternatives.

Returning to the modeling framework, regression of quantiles [25] or expectiles [26] instead of the mean–variance pair is another plausible option to approximate a distribution. The *conditional autoregressive value at risk* (CAViaR) [27] and the *conditional autoregressive expectile* (CARE) [28] models stand out as the most widely referred quantile and expectile regression models, respectively, in the economics literature. Thus, we find it worthwhile to also explore these approaches.

To the authors' knowledge, no previous work has yet fully treated GARCH models in the context of probabilistic electricity load forecasting. Among the extremely few prior studies, [29–31] considered several variants of the GARCH model in point forecasting, whereas [32] evaluated the calibration of some upper quantiles without incorporating also the sharpness of them via a proper rule such as the tick (pinball) loss function [33]. In this work, we employ proper rules and thereby provide a complete probabilistic treatment to demonstrate the performance gains brought by the GARCH model.

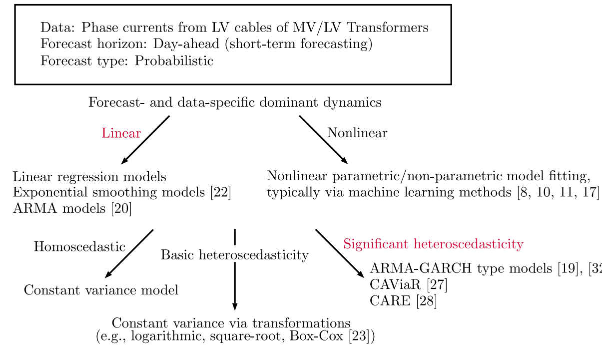

Fig. 1. The problem context and decision mechanism in modeling.

The very recent works [2] and [8] also concern probabilistic forecasting, but now in a nonlinear setting based on additive models, with [8] covering quantile regression as well in order to compete with the mean–variance model. The same trend of having only a few papers in the literature also holds for the application of quantile and expectile regression in demand forecasting. In particular, [9] and [34] employ the former in the context of forecast averaging and multiple quantile regressions for nonparametric modeling, respectively, and [35] uses both approaches to propose a reduced dimensional method with the principle component dynamics described by vector ARX models. All these recent publications evince the growing research effort being directed towards the topic that is also of our interest. However, our analysis differs from these studies in terms of focusing on the day-ahead forecasts on low voltage transformer cables where the use of GARCH models is sufficient to cope with the uncertainty. As another difference, we search for and propose an accurate and fast forecasting scheme, with an iterative estimation mechanism placed at the core of it.

Summarizing all these contributions, the novelty of this paper is threefold: full probabilistic treatment of the GARCH model in load forecasting, its application on currents of low voltage transformer cables, and its utilization in a practical prediction mechanism (i.e., the ILS$_c$ method in Section 5.2) that selects the best model parameters with respect to the proper scoring rule (performance metric) on which forecasts are evaluated. Therefore, different from other forecast methods, this metric (i.e., the CRPS or the QS in Section 5.1) is directly involved in the determination of the ultimate model.

The rest of this paper is organized as follows. In Section 2, the mathematical framework that will aid in the development of our ideas is introduced. Then in Section 3, the proposed conditional mean–variance model is elaborated, with a special attention given to estimation methods. Section 4 describes the CAViaR and CARE models and their application-specific use. Moreover, comparative performance evaluations based on empirical results are presented in Section 5, and finally, the drawn conclusions are recapitulated in the last section.

2. Mathematical preliminaries

All models in this paper make use of the ARMAX model structure that can concisely be expressed by means of the function $g_*(\boldsymbol{ar}_{t^-}, \boldsymbol{ma}_{t^-}, \boldsymbol{x}_t)$ defined as

$$c_* + \sum_{i\in\mathcal{I}_*} \alpha_{*,i} ar_{t-i} + \sum_{j\in\mathcal{J}_*} \beta_{*,j} ma_{t-j} + \sum_{\ell\in\mathcal{L}_*} \gamma_{*,\ell} x_{\ell,t} \tag{1}$$

where "$*$" is a model-specific identifier, t is the discrete-time index, c_*, $\{\alpha_{*,i}\}_{i\in\mathcal{I}_*}$, $\{\beta_{*,j}\}_{j\in\mathcal{J}_*}$, $\{\gamma_{*,\ell}\}_{\ell\in\mathcal{L}_*}$ are the involved parameters, $\mathcal{I}_*$, $\mathcal{J}_*$, and $\mathcal{L}_*$ are subsets of positive integers indicating