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Adaptive traffic signal control with equilibrium constraints under stochastic demand

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A B S T R A C T

This study develops a methodology to model transportation network design with signal settings in the presence of demand uncertainty. It is assumed that the total travel demand consists of commuters and infrequent travellers. The commuter travel demand is deterministic, whereas the demand of infrequent travellers is stochastic. Variations in demand contribute to travel time uncertainty and affect commuters' route choice behaviour. In this paper, we first introduce an equilibrium flow model that takes account of uncertain demand. A two-stage stochastic program is then proposed to formulate the network signal design under demand uncertainty. The optimal control policy derived under the two-stage stochastic program is able to (1) optimize the steady-state network performance in the long run, and (2) respond to short-term demand variations. In the first stage, a base signal control plan with a buffer against variability is introduced to control the equilibrium flow pattern and the resulting steady-state performance. In the second stage, after realizations of the random demand, recourse decisions of adaptive signal settings are determined to address the occasional demand overflows, so as to avoid transient congestion. The overall objective is to minimize the expected total travel time. To solve the two-stage stochastic program, a concept of service reliability associated with the control buffer is introduced. A reliability-based gradient projection algorithm is then developed. Numerical examples are performed to illustrate the properties of the proposed control method as well as its capability of optimizing steady-state performance while adaptively responding to changing traffic flows. Comparison results show that the proposed method exhibits advantages over the traditional mean-value approach in improving network expected total travel times.

1. Introduction

Traffic signal control is an important form of traffic control and management to improve urban traffic operations. Any changes in the signal settings, however, will trigger travellers to modify their routing plans in response. Hence, it is essential to include the effects of route choices to determine any signal control improvement scheme for network-wide operations. Many traffic control systems, such as RHODES (Head et al., 1992) and UTOPIA (Mauro and Di Taranto, 1989), incorporate network signal control at the strategic level of a hierarchical control structure, to take into account the influence of travellers' route choice behaviour. From a network planning point of view, network signal control is able to trigger better equilibrium flow patterns associated with better network performance. As such, network signal control is one instance of the network design problem (NDP) in determining optimal signal settings (Chiou, 2008; Yang and Jayakrishnan, 2015; Huang et al., 2016, 2017). Numerous research efforts have been made on

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developing models and algorithms for the NDP with a deterministic approach, assuming that traffic states are certain and perfectly known to planners and travellers. Comprehensive overviews on the NDP are provided in Magnanti and Wong (1984), Yang and Bell (1998), Chiou (2005), and Farahani et al. (2013).

In practice, road networks experience uncertainty due to variations in traffic demand and supply, resulting from day-to-day demand fluctuations, special events, incidents, etc. Applying deterministic approaches that ignore the uncertainty effects may lead to inaccurate evaluations on the network performance, and hence suboptimal control decisions. To overcome the shortcomings of the deterministic approaches, it has led to the development of reliable NDP models under uncertainty. Chen and Yang (2004) formulated two stochastic NDP models that consider spatial equity and demand uncertainty: an expected-value model that minimizes the expected total travel time subject to maximal spatial equity ratio, and a chance-constrained model that formulates the equity constraint as a probabilistic constraint. Chootinan et al. (2005) introduced a new capacity-reliability index to measure the probability that all network links are operated below their capacities and provided a reliability-based network design problem to maximize the capacity-reliability index under stochastic user equilibrium constraints. Chen et al. (2006) examined further the capacity-reliability index and formulated a bi-level programming to determine signal settings for minimizing the probability of link failures for a signal-controlled road network.

This study focuses on network signal design taking into account demand uncertainty. Different from the aforementioned studies that capture the impact of stochastic variations merely in the upper-level performance function, this paper also incorporates this impact into travellers' route choice behaviour and the lower-level user equilibrium problem. We consider that the total travel demand consists of two parts, commuters and infrequent travellers. Commuters travel regularly on a daily basis and the demand of commuters is fairly stable, whereas the demand of infrequent travellers, e.g., tourists who make irregular trips, is stochastic. It is assumed that these two groups of people behave differently in their route choice decisions. Commuters would habituate the choice decisions based on their experienced travel time and settle into a long-term traffic equilibrium. Infrequent travellers are not fully aware of the alternatives and choose among a limited set of routes with fixed proportions.

Due to the stochastic demand of infrequent travellers, travel time becomes uncertain. It is known that travel time variability plays a crucial role in travellers' route choice behaviour (Jackson and Jucker, 1981). The empirical study of Abdel-Aty et al. (1995) reported that travel time variability is one of the most important factors in route choice considerations. Commuters generally take into account the randomness in travel time in their route choice decisions. In order to capture uncertainty in traffic equilibrium modelling, Watling (2006) introduced a generalised form of the user equilibrium model to reflect travellers' aversion to the risk of late arrival in the face of uncertain travel time. As travellers generally add a travel time margin to avoid late arrivals, a notion of travel time budget was proposed (Lo et al., 2006; Siu and Lo, 2008), defined as the summation of expected travel time and a travel time margin. The route choice behaviour in the face of uncertain travel time in degradable networks is then characterized with a travel time budget equilibrium, in which all selected routes have the minimal travel budget, while all unused routes have an equal or longer travel time budget. This study applies the concept of travel time budget equilibrium to the network signal design problem. A reliable network signal design that explicitly captures travellers' risk aversion under demand uncertainty is then developed.

In the presence of demand uncertainty, it is quite natural to introduce a *control buffer*, defined as the excess green time to the optimal control plan that is calculated with pure deterministic demand, to hedge against demand variability (Lo, 2006). On the one hand, the control buffer benefits the traffic system performance by reducing the chance of overflow or oversaturation; on the other hand, it will bring in certain conservatism in a sense that it may trigger commuter re-routing behaviour and change the equilibrium flow patterns. To account for both transient and commuter equilibrium performance in the presence of demand variations, this paper introduces a two-stage stochastic program to formulate the network signal design problem. Under the two-stage stochastic program, we have a set of control decisions to be taken, which is determined based on the stochastic demand distribution before the actual demand realization. These decisions are called first-stage decisions. Later, upon realizations of the random demand, the actual arrivals are used to determine the second-stage corrective or recourse actions. Recently, the two-stage stochastic program has been applied to adaptive signal settings, for isolated traffic signal control (Tong et al., 2015), as well as coordinated signal control (Ma et al., 2016; Li et al., 2018) under random arrivals. These control schemes are applied to manipulate traffic operations without considering re-routing effects. This study further incorporates the equilibrium constraints in the two-stage optimization framework. The optimal control policy derived under the two-stage framework is able to (1) optimize the steady-state network performance in the long run, and (2) respond to short-term demand variations. In the first stage, a base control plan with a buffer against variability is introduced to control the equilibrium flows and the resulting steady-state performance. Upon realizations of the random demand, adaptive signal settings are then determined in the second stage to address the occasional demand overflows, so as to avoid transient congestion and spillback. The overall objective is to minimize the expected total travel time across the random scenarios.

The proposed two-stage stochastic approach can be useful as a planning tool. In contrast to the traditional approach for network signal design that usually assumes deterministic demand and hence ignores the uncertainty effect, the two-stage stochastic approach determines a reliable network signal design, which reserves a *built-in* control buffer to hedge against uncertainty. The base control plan with the control buffer appropriately designed will not induce undesirable routing behaviour, for instance rat running in which commuters randomly take alternatives resulting in a chaotic system. After implementation of the base strategic control plan, signal settings are adjusted to address transient overflows that cannot be covered by the control buffer. Whereas the traditional deterministic approach is a *one-off* control policy, the two-stage stochastic program determines a control strategy with *recourse decisions*. More specifically, it determines a fixed base control plan, which maintains a certain sense of predictability of the control plan and hence would not encourage erratic route choice responses, while occasionally adapts the green time allocation on a need basis to address demand overflows.