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Optimal switched control design for automatic train regulation of metro lines with time-varying passengers arrival flow

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ABSTRACT

This paper investigates the optimal switched control design problem for automatic train regulation of metro lines with time-varying passengers arrival flow. The variation of passenger arrival flow is described by an uncertain switched linear function. A new train traffic model of the metro-line operation is proposed based on an uncertain switched hybrid system. According to the stability analysis method of the switched system, a sufficient condition for the existence of state-feedback control as automatic train regulation strategy is given in terms of matrix inequalities, which ensures the stability of metro line system under a disturbance or a disruption. Moreover, a nonlinear optimal control problem is formulated to determine the optimal switched control design as automatic train regulation to minimize the train delays and control cost. The formulated nonlinear optimal control problem is converted into a convex optimization problem involving linear matrix inequalities, which can be solved in polynomial time and has low computation burden. Numerical examples are given to illustrate the effectiveness of the proposed methods.

1. Introduction

With the increasing traffic congestion in modern large cities, urban metro transportation system has became an attractive mode of transports to relieve the traffic pressure, which has its inherent features of reliability, punctuality, energy conservation and high-capacity (Mannino and Mascis, 2009; Wang et al., 2015). Because of the accumulation of passengers on the metro line, especially during the peak hours, if one train is delayed by a disturbance or a disruption, the train delay will be increased from one station to the next station. Therefore, high-frequency metro line system is naturally unstable (Van Breusegem et al., 1991; Fernandez et al., 2006). This instability will bring the delay propagation of trains along stations and meanwhile increase the waiting time of passengers, thereby negatively affecting the operation efficiency of metro line. In case a disturbance or a disruption occurs, an efficient train regulation strategy, by adjusting the running time and the dwell time of each train, is therefore necessary to prevent the instability and improve the operation efficiency of metro lines.

Train regulation problems for recovering the delays can be broadly classified into three categories in the existing literature. (1) For the situation with small delays, each train can recover its own delay by using the time margins (buffer time) (Vansteenwegen and Oudheusden, 2004; Abril et al., 2008). (2) For the situation with large delays, the affected trains will need several stations to compensate the delays, then a transient period is needed to reach the nominal timetable. In this case, the regulation strategy tries to minimize mainly timetable deviations during the transient, which is for a full timetable recovery (Fernandez et al., 2006; Lin and Sheu, 2011). (3) For the more larger delays, if the duration of the regulation transient and the magnitude of time deviations from nominal timetable are unacceptable, then a re-scheduling process is needed, and a new delayed nominal timetable could be

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established (Corman et al., 2012; Dundar and Sahin, 2013; Xu et al., 2016). Specifically, consider a metro line for which the slack time is 15 s, the maximum adjustment for each train at one station is 30 s and the acceptable regulation transient is 10 stations. Then if the train delay is less than 15 s, it can recover its own delay by using the time margins, which belongs to problem (1) with small delays. If the train delay is larger than 15 s and less than $30\ \text{s} \times 10 = 300\ \text{s}$, the affected train will need several stations to compensate the delays, then a transient period is needed to reach the nominal timetable, which belongs to problem (2) with large delays. Otherwise, if the train delay exceeds 300 s, the duration of the regulation transient and the magnitude of time deviations from nominal timetable are unacceptable, then a re-scheduling process is needed, which belongs to problem (3) with more larger delays. For the above three train regulation problems, this paper focuses on the problem (2) with large delays in which the regulation strategy tries to compensate completely the delays with respect to a nominal timetable in a metro line system.

For the train regulation problems, the buffer times or supplements are usually designed in nominal dwell time and running time to absorb the train delays. However, the buffer time allocation is static, which cannot be used dynamically and flexibly from a system-wide point of view. This may reduce system capacity utilization for the possible redundant buffer time. Automatic train regulation (ATR) is a core function of modern metro signalling systems, which is used to recover train delays resulting from disturbances by dynamically adjusting the running time and dwell time of each train, thereby reducing the potential redundant buffer time and improving the system capacity utilizations. Recently, many efficient automatic train regulation methods have been proposed for metro lines (Van Breusegem et al., 1991; Goodman and Murata, 2001; Chang and Chung, 2005; Lin and Sheu, 2011).

The early work by Van Breusegem et al. (1991) proposed a complete discrete dynamic traffic model for high-frequency metro lines and analyzed the natural instability of metro lines. The state-feedback control algorithms were designed to ensure the stability of metro line system and the minimization of a given performance index. Followed by the discrete dynamic traffic model, Fernandez et al. (2006) further proposed a predictive traffic regulation model for metro loop lines to compensate timetable and headway deviations, and the proposed quadratic programming model can be solved efficiently in real time. Lin and Sheu (2010) proposed an automatic train regulation design strategy by using a heuristic dynamic programming method. The proposed automatic train regulation system was easier to handle the non-linear and stochastic characteristics of metro line and a near-optimal regulation can be obtained more rapidly and accurately. Moreover, by considering the energy consumption, Sheu and Lin (2012) designed an automatic train regulation strategy with energy saving via coasting and station dwell control based on a dual heuristic programming method. The evaluation of the automatic train regulation design was conducted with field data, which showed that better traffic regulation with higher energy efficiency is attainable. Li et al. (2016) applied a robust model predictive control method for automatic train regulation design in underground railways to ensure the minimization of an upper bound on the metro system cost function, which showed that the proposed train regulation has a low online computation burden. This study is also based on the discrete dynamic traffic model adopted in Van Breusegem et al. (1991), Fernandez et al. (2006), Lin and Sheu (2010), and Li et al. (2016) to further investigate the train regulation problem for metro lines.

In the most existing literature, the passenger arrival flow at the stations is usually assumed to be a constant or pre-known variable (Van Breusegem et al., 1991; Fernandez et al., 2006; Lin and Sheu, 2010; Niu and Zhou, 2013; Sun et al., 2014). In practice, the passenger arrival flows are not accurately known and dynamically changing with time. Consider the fact that the accumulation of passengers lead to the instability of metro lines, which has a significant effect on the operation efficiency of metro lines. It requires more accuracy for the time-varying passenger arrival flow of metro lines in practice, especially during the peak hours. A common approach to deal with this uncertainty on the passenger arrival flow is to assume a probability distribution on the arrival rates (Yin et al., 2016). However, it is difficult in practice to determine such probability distributions and the probability distributions cannot accurately denote the arrival rates. Therefore, it may be more realistic to assume a range of variation on the passenger arrival rate rather than a probability distribution (Adida and Perakis, 2010). Moreover, the passenger arrival rates are significantly different for the different periods of one day. For example, the passenger arrival rate during the rush hours is larger than that during the off-peak hours. Thus, to further describe the time-varying passenger arrival flow more accurately, it is necessary to apply different ranges of variation to denote the passenger arrival rates on the different periods of one day. To address this problem, we adopt an uncertain switched linear function to describe the time-varying passenger arrival flow on one day, where the switched modes represent the different periods of one day and for each switched mode, the passenger arrival rates are denoted by a range of variation. Then the variation of passenger arrival flow can be approximated by a uncertain switched linear function. The train traffic model of the metro-line operation is thus formed as a uncertain switched hybrid system. The switched hybrid system theory has been studied in the literature (Daafouz et al., 2002; Lin and Antsaklis, 2009; Hajiahmadi et al., 2015), which can be well applied to study the train traffic model with the switched hybrid dynamic.

Motivated by the above discussions, in this paper, we propose a new train traffic model of the metro-line operation based on a switched hybrid system. The passenger arrival rates at different periods have different switched modes and for each switched mode, the passenger arrival rates are denoted by a range of variation rather than a probability distribution. The goal of the paper is to design the optimal state-feedback control law as automatic train regulation to minimize the train delays and control cost for a metro line system under a disturbance or a disruption. Based on stability analysis for the switched hybrid system, a sufficient condition for the existence of state-feedback control as the train regulation strategy is given in terms of matrix inequalities, which ensures the stability of metro line system. Moreover, a nonlinear optimal control problem is formulated to determine the optimal switched control as automatic train regulation to minimize the train delays and control cost. Consider the fact that existing dynamic programming methods become computationally prohibitive to deal with the optimal control problem for the uncertain switched hybrid system. To overcome this difficulty, the nonlinear optimal problem is converted into a convex optimization problem involving linear matrix inequalities (LMIs). The LMIs-based optimization problem can be solved in polynomial time, which has a low computation burden and can be implemented on-line, satisfying the real-time requirement for the train regulation problem. In addition, the so-called