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Second order macroscopic traffic flow model validation using automatic differentiation with resilient backpropagation and particle swarm optimisation algorithms

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A B S T R A C T

The problem of validating the Modéle d'Écoulement de Trafic sur Autoroute NETworks (METANET) model of a motorway section is considered. Model calibration is formulated as a least squares error minimisation problem with explicit penalisation of fundamental diagram parameter variation. The Automatic Differentiation by Overloading in C++ (ADOL-C) library is incorporated into the METANET source code and is coupled with the Resilient Back Propagation (RPROP) heuristic for solving the minimisation problem. The result is a very efficient system which is able to be calibrate METANET by determining the density and speed equation parameters as well as the fundamental diagrams used. Information obtained from the system's Jacobian provides extra insight into the dynamics showing how sensitivities propagate into the network. A 22 km site near Sheffield, UK, using data from three different days is considered. In addition to the ADOL-C/RPROP system, three particle swarm optimisation algorithms are used for solving the calibration problem. In all cases, the optimal parameter sets found are verified on data not used during calibration. Although, all three sets of data display a similar congestion pattern, the verification process showed that only one of them is capable of leading to parameter sets that capture the underlying dynamics of the traffic flow process.

1. Introduction and background

The macroscopic description of traffic along a motorway was introduced in the seminal papers of Lighthill and Whitham (1955) and Richards (1956), resulting to the LWR model relating the density ρ and flow q at location s and time t

$$\frac{\partial\rho(s,t)}{\partial t}+\frac{\partial q(s,t)}{\partial s}=0. \tag{1}$$

The nonlinear relationship between flow and density is

$$q(s,t)=\rho(s,t)V[\rho(s,t)] \tag{2}$$

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where $V[\rho(s,t)]$ (km/h) is the equilibrium relationship between density and mean speed, i.e. the Fundamental Diagram (FD) of traffic. For any given site a number of FD may be used reflecting changes in the number of lanes and road geometry, bottleneck locations and significant gradient variability.

Typically, a space and time discretised version of Eq. (1), along with the FD constitute the basic elements of a first order macroscopic traffic flow model, see e.g. Lighthill and Whitham (1955), Richards (1956), and Daganzo (1994).

Payne-Whitham type second order models result from coupling Eq. (1) with an empirical equation governing the mean speed $v(s,t)$ dynamics (Payne, 1971; Whitham, 1974), with the form

$$\frac{\partial v(s,t)}{\partial t} + v(s,t)\frac{\partial v(s,t)}{\partial s} + \frac{1}{\rho(s,t)}\frac{\partial P(s,t)}{\partial s} = \frac{1}{\tau}\{V[\rho(s,t)] - v(s,t)\} \tag{3}$$

where τ is a relaxation constant and $P(s,t)$ a pressure term, which gives rise to a range of different models of this family (Helbing et al., 2002).

Irrespective of the model's order, a number of parameters characterising the aggregate infrastructure-vehicle-driver behaviour are used. Using data sets of traffic counts and vehicle speeds, typically obtained by means of inductive loop detectors embedded in the motorway, a rigorous model validation procedure needs to take place, for identifying an optimal set of parameters. This procedure is performed in two steps, calibration and verification. Calibration requires the solution of an error minimisation problem and verification involves testing the solutions obtained on data that were not used for furnishing the corresponding calibration optimisation problem. This paper is concerned with model validation of a traffic flow model along the lines given in Cremer and Papageorgiou (1981) and Papageorgiou (1983).

First order traffic flow model calibration is concerned mostly with determining the FD parameters of discrete road sections, depending on the discretisation scheme used for (1). The most commonly used model of this order is the Cell Transmission Model (CTM) by Daganzo (1994) and detailed calibration efforts may be found in Munoz et al. (2004, 2006).

A comparative study of the CTM and the second order Modéle d'Écoulement de Trafic sur Autoroute NETworks (METANET) by Messmer and Papageorgiou (1990) for a motorway in Greece based on the Nelder-Mead algorithm (Nelder and Mead, 1965) is provided by Spiliopoulou et al. (2014).

An extensive validation study of METANET, using a source code different than the one employed here, for the Paris ring road is reported by Papageorgiou et al. (1990). The METANET validation of the large scale network of the Amsterdam orbital motorways is described in Kotsialos et al. (1998, 2002). In Frejo et al. (2012) a METANET model parameter identification algorithm is discussed using data from a 4.65 km stretch of a California highway; the original expression used for FD in METANET is replaced with a two-regime model and the resulting optimisation problem is solved using a sequential quadratic programming algorithm.

A linear varying parameter method is described by Luspay et al. (2009, 2010, 2011) for identifying second order model parameters. A simultaneous perturbation stochastic approximation method is used by Alessandri et al. (2006) for the same purpose using information from mobile phones. In Treiber and Kesting (2012) a method calculating the model parameters by comparing the congestion pattern of the data and model output aiming at avoiding incorrect data forms, was used. The model used by Treiber and Kesting (2012) was validated by Ngoduy and Maher (2012) on a 10 km section of a UK highway.

As is mentioned by Ngoduy and Maher (2012), the optimisation problem related to model calibration has numerous local minima. Hence, efficient optimisation algorithms need to be used for obtaining parameter sets that make models capable of representing traffic dynamics. Most of the proposed, if not all, algorithms used are population based derivative free methods, employing direct or stochastic search. In a recent overview of nonlinear programming methods used for macroscopic traffic flow model calibration by Kontorinaki et al. (2015), gradient based optimisation algorithms are not considered as a viable option due to the nonlinear and non-convex nature of the least-squares optimisation problem. However, it is shown here that a simple globalisation strategy based on a multistart scheme of a gradient based heuristic is capable of efficiently solving this problem.

For the modelling of the Paris and Amsterdam sites discussed by Papageorgiou et al. (1990) and Kotsialos et al. (1998, 2002), the deterministic search algorithm of Box (1965) was used. A cross entropy method is used by Ngoduy and Maher (2012). A simplex based algorithm was used by Ngoduy et al. (2004) to validate various numerical schemes used for solving the macroscopic model equations. A number of population based derivative free optimisation algorithms used for calibration are discussed by Spiliopoulou et al. (2015).

A nonlinear mixed integer optimisation formulation was introduced for the macroscopic traffic flow model calibration problem which was solved by means of a genetic algorithm by Poole and Kotsialos (2012). METANET was treated as a simulation black box. An additional requirement introduced was the automatic spatial assignment, i.e. determination of the location and extension, of fundamental diagrams (FD). The motivation behind this is that current calibration practice either uses expert engineering opinion to make a decision about the FD or use a separate FD for every discrete road segment. In the first case, intuition, past experience, visual inspection and preliminary data analysis result to an *ad-hoc* approach leading away from systems that embed knowledge in their own structure and the display of more intelligent forms of automation (Kotsialos and Poole, 2013, 2016). In the latter case, overparametrisation is a clear risk since typically three parameters are necessary for defining a FD.

The problem formulation suggested by Poole and Kotsialos (2012) allows the selection of FD location for homogeneous road stretches, which themselves are split into segments. It also penalises the variance between their parameters. The