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# Integrating capacity analysis with high-speed railway timetabling: A minimum cycle time calculation model with flexible overtaking constraints and intelligent enumeration

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ABSTRACT

Compared with most optimization methods for capacity evaluation, integrating capacity analysis with timetabling can reveal the types of train line plans and operating rules that have a positive influence on improving capacity utilization as well as yielding more accurate analyses. For most capacity analyses and cyclic timetabling methods, the cycle time is a constant (e.g., one or two hours). In this paper, we propose a minimum cycle time calculation (MCTC) model based on the periodic event scheduling problem (PESP) for a given train line plan, which is promising for macroscopic train timetabling and capacity analysis. In accordance with train operating rules, a non-collision constraint and a series of flexible overtaking constraints (FOCs) are constructed based on variations of the original binary variables in the PESP. Because of the complexity of the PESP, an iterative approximation (IA) method for integration with the CPLEX solver is proposed. Finally, two hypothetical cases are considered to analyze railway capacity, and several influencing factors are studied, including train regularity, train speed, line plan specifications (train stops), overtaking and train heterogeneity. The MCTC model and IA method are used to test a real-world case involving the timetable of the Beijing–Shanghai high-speed railway in China.



## 1. Introduction

Periodic train timetables define the planned departure and arrival times of trains at stations with respect to a cycle time. They are predominantly used for passenger railways because they offer many advantages, such as regular train connections, which ensure high-quality transfer service. With the growth of railway passenger ridership, many train operating companies have shown great interest in studying and increasing railway capacity. In most such studies, capacity is typically defined as the maximum number of trains that can be run on a given infrastructure for a certain timetable in a fixed period of time (Abril et al., 2005). However, railway capacity varies with changes not only in infrastructure but also in train line plans and operating rules. With the gradual completion of the high-speed railway network in China, the possibilities for further updates to the infrastructure are becoming more limited; therefore, there is an urgent need to investigate which types of train line plans and operating rules are most beneficial for capacity utilization based on a fixed infrastructure. Thus, it is necessary to analyze capacity based on various factors that influence operation management, such as train line plans and

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operating rules. Capacity analysis integrated with timetabling can satisfy this requirement because train line plans and operating rules are closely associated with timetables. The most relevant classes of methods for capacity analysis are analytical methods, optimization methods, and simulation methods (Abril et al., 2008). In our opinion, integrating capacity analysis with timetabling results in a more accurate (realistic) treatment of the problem than analytical methods and is easier (requires fewer inputs) than simulation methods. Compared with other optimization methods, integrating capacity analysis with timetabling can reveal the relationship between capacity and specific operating rules. Moreover, various line plans and operating rules lead to different levels of transport service quality, and the trade-off between capacity and quality is complex and needs to be studied. The objective of future optimization is not to maximize capacity at the expense of quality but rather to improve both measures synchronously (Rao et al., 2015).

In this paper, we demonstrate that cyclic timetabling models with the objective of a minimum cycle time are highly applicable in capacity analysis. We believe that integrating capacity analysis with timetabling is a promising means of studying the trade-off between operating rules and railway capacity, which is the main objective and first contribution of this paper. The second contribution of this paper is that we construct new constraints, including a ***non-collision constraint*** and ***flexible overtaking constraints*** (FOCs), based on the variation of the modulo variables in the periodic event scheduling problem (PESP). This method of constraint construction is distinct from those applied in previous research because the constraints built using this new approach are more concise and precise. Meanwhile, the relationship between the modulo variables and overtaking is revealed. Because of the complexity of the PESP and based on the characteristics of our model, we also propose a heuristic method for integration with the CPLEX solver to reduce the computation time, which is the third contribution of this paper.

The remainder of this paper is organized as follows. A literature review is presented in Section 2. In Section 3, we first define our constraint graph (on which the proposed ***minimum cycle time calculation*** **(MCTC)** model is based) and then describe the MCTC model, the non-collision constraint and FOCs. Section 4 describes the proposed ***iterative approximation*** (IA) method that is integrated with the CPLEX solver, and we present various capacity analyses of hypothetical cases and computational results for a real-world case in Section 5. Finally, conclusions are presented in Section 6.

## 2. Literature review

In recent years, many remarkable studies have been devoted to train timetabling (e.g., Goverde, 2010; Harrod, 2012; Niu and Zhou, 2013; Arenas et al., 2015; Schmidt and Schöbel, 2015), train scheduling (e.g., Lindner, 2000; Törnquist, 2006; Harrod, 2011; Corman et al., 2012; Sun et al., 2014; Wang et al., 2015), train operation management (e.g., Caprara et al., 2007; Goverde and Meng, 2011; Meng and Zhou, 2011; Goverde and Hansen, 2013), and related algorithms (e.g., Bertacco et al., 2007; Zhou and Zhong, 2007). In particular, Zhou and Zhong (2005) studied a double-track train-scheduling problem with multiple objectives to minimize both the expected waiting times for high-speed trains and the total travel times of high-speed and medium-speed trains. In their model, overtaking between trains at stations is discussed and accurately described by constraints. However, their research was based on non-cyclic timetabling. Among the research performed on cyclic train timetabling, models based on the PESP, which was introduced by Serafini and Ukovich in 1989 (Peeters, 2003), have demonstrated great power in periodic railway timetabling. A PESP-based model for the cyclic railway timetabling problem (CRTP) was first considered in 1993, and a stronger model, the cycle periodicity formulation (CPF), was introduced in 1999 (Peeters, 2003). The PESP and CPF are based on the construction of an auxiliary graph, whose nodes correspond to events (train departures and arrivals) and whose arcs model the constraints acting on the time separations between those events (Cordone and Redaelli, 2011). This auxiliary graph, known as the *event-activity network* (EAN), which is also used in this paper, has been widely applied in the literature on train timetabling (e.g., Kroon and Peeters, 2003; Schöbel, 2007; Liebchen et al., 2010; Schachtebeck and Schöbel, 2010).

Many extended models and effective algorithms based on the PESP have been studied in depth in recent years (e.g., Kroon and Peeters, 2003; Liebchen, 2004; Mathias, 2008; Xie and Nie, 2009; Caimi et al., 2011; Cordone and Redaelli, 2011; Kroon et al., 2013). With regard to operating rule constraints, Peeters (2003) and Caimi et al. (2011) discussed a *non-collision constraint* with a variable trip time to prevent overtaking between successive stations. Moreover, Kroon and Peeters (2003) and Liebchen and Möhring (2007) proposed subdividing an initial trip arc into new smaller ones to satisfy the requirements of the PESP framework. Regarding the overtaking relationships of trains at stations, the corresponding constraints are typically constructed based on the lower and upper bounds of running time; however, in this paper, we prefer to describe these relationships in a simpler yet accurate manner. With regard to the objective function, an objective for the PESP based on the *minimum cycle time* $T$ (i.e., the minimum period length of one regular timetable) was presented by Sparing and Goverde (2013), where the stability of the timetable is considered because the relationship between the nominal cycle time and the minimum cycle time $T$ can describe the degree of capacity utilization represented by the timetable (Hansen and Pachl, 2008). Regarding applicable algorithms, Siebert and Goerigk (2013) studied a series of experimental comparisons of various extended PESP models (the Origin Destination aware PESP (ODPESP) and the Extended PESP (EPESP)) and three different methods based on the modulo simplex algorithm proposed by Nachtigall and Opitz (2008), which is a powerful heuristic for solving the PESP (Goerigk and Schöbel, 2013). For an in-depth overview of the PESP, CRTP, and CPF, we refer to Peeters (2003) as well as Liebchen (2004), Liebchen and Möhring (2007) and Liebchen et al. (2010).