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Sharp probability inequalities for reliable evacuation planning

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ABSTRACT

This paper contributes to the literature by providing new techniques to determine optimal evacuation routes when limited information is known about the evacuation demand and road capacities. Specifically, this research complements prior work by requiring only their first and second moments to be known. The inclusion of an optional symmetry assumption is rigorously shown to further reduce the evacuation time. Finally, we demonstrate that the probability inequalities that serve to ensure the confidence level of the evacuation plan are sharp. A case study illustrates the core idea of demand inflation and supply deflation in obtaining reliable evacuation plans.

1. Introduction

Natural and man-made disasters in recent years have highlighted the need for effective evacuation planning models. This need is also reflected by the increasingly larger number of research publications in this area, ranging from empirical studies on the evacuation behavior of evacuees (e.g. see Ng et al., 2014, 2015) to shelter allocation decision support systems (e.g. see Kongsomsaksakul et al., 2005; Ng et al., 2010; Kulshrestha et al., 2011; Li et al., 2012; Goerigk et al., 2014), evacuation network design problems (e.g. see Ng and Waller, 2009; Xie et al., 2010), alternative evacuation modes (e.g. see Kulshrestha et al., 2014; Zheng, 2014), emergency logistics (Sheu, 2007, 2010) and the determination of optimal evacuation routes, which is the focus of the current paper.

Early models to determine optimal evacuation routes make the assumption of determinism: Evacuation demand as well as road capacities are assumed to be known with certainty during evacuations (e.g. see Tuydes and Ziliaskopoulos, 2006; Liu et al., 2006; Chiu et al., 2007; Chiu and Zheng, 2007 and the references therein). To relax this assumption, a limited number of researchers then started to model demand and supply factors as random variables. For example, Yazici and Kaan (2007) and Lim et al. (2015) employed conventional chance constraint programming to model uncertainties in road capacities during evacuations, assuming that explicit probability distributions describing the random road capacities are known. Li and Ozbay (2014) took on a different perspective by explicitly modeling road capacity uncertainty as an endogenous risk of the transportation system.

To eliminate the need for complete probability distributions, which might not be available in practice, Ng and Waller (2010) proposed an intuitive framework that only required the knowledge of the mean values and upper and lower bounds on the evacuation demand and road capacities. To account for uncertainty in evacuation demand, they suggested planning for an inflated evacuation demand. On the other hand, to account for uncertain road capacities during evacuations, they

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showed that one can plan for deflated/ reduced road capacities. The amount of demand inflation and capacity deflation necessary to ensure an *a priori* confidence level were given by analytical expressions. While the framework was novel and intuitive, one of the drawbacks of the model was that the amount of inflation and deflation might be higher than strictly necessarily (Ng and Waller, 2010). In other words, the derived probability inequalities could not be shown to be sharp, with the possibility of leading to unnecessarily conservative evacuation plans. In this paper, we revisit this intuitive idea of demand inflation and capacity deflation. Specifically, we fundamentally advance this framework by presenting new and complementary probability inequalities for the case when only the means and variances of the uncertain quantities are known. Moreover, unlike in Ng and Waller (2010), the derived inequalities are shown to be sharp.

The remainder of this paper is organized as follows. In Section 2, we briefly review an existing evacuation model that the current paper builds on. New inequalities are derived in Section 3 that form the main results in this paper. A case study is given in Section 4. Finally, Section 5 summarizes and concludes this paper.

2. Model review

Before we present the main results in this paper, let us first briefly review a variation of an accepted evacuation optimization model in the literature, e.g. see Tuydes and Ziliaskopoulos (2006), Chiu and Zheng (2007), Ng and Waller (2010). This model employs the cell transmission model (CTM) to accurately model traffic dynamics (Ziliaskopoulos, 2000). In the cell transmission model, road networks are discretized into cells. At each time step, vehicles move to the next cell, if space is available. Otherwise, they remain at the current cell. Real world traffic phenomena such as shockwaves and link spillover can be accurately modeled in a CTM framework. For more details of this widely popular traffic flow model, please see, for example, Daganzo (1994, 1995) and Ziliaskopoulos (2000).

Sets

C set of all cells
C_S sink cell, i.e. evacuation destination
C_R set of source cells, i.e. all cells with evacuation demand
T set of disjoint time intervals covering the planning period; $|T|$ denotes its cardinality
E set of cell connectors
E_S set of sink cell connectors
$\Gamma(i)$ set of cell connectors emanating from cell i
$\Gamma^{-1}(i)$ set of cell connectors incident to cell i

Parameters

ζ_i initial number of vehicles in cell i
n_i^t maximum possible number of vehicles in cell i at time t (jam density)
D_i^t evacuation demand at cell i at time t (a random variable)
Q_i^t maximum number of vehicles that can flow into or out of cell i at time t (a random variable)
β_i^t maximum permissible probability that resulting evacuation plan cannot meet demand at cell i at time t
γ_i^t maximum permissible probability of violating constraints (6) and (7) at cell i at time t
δ_i^t ratio of forward and backward propagation speed for cell i at time t (assumed to be equal to 1 in this paper)

Decision variables

x_i^t number of vehicles in cell i at time t
y_{ij}^t number of vehicles moving from cell i to cell j at time t

2.1. Objective function and constraints

$$\underset{x,y}{\text{minimize}} \sum_{(i,j)\in E_S} \sum_{t\in T} t \cdot y_{ij}^t \tag{1}$$

subject to

$$\Pr\left\{x_i^t - x_i^{t-1} + \sum_{(i,j)\in\Gamma(i)} y_{ij}^{t-1} > D_i^t\right\} \geqslant 1 - \beta_i^t \quad \forall i \in C_R, \forall t \in T \tag{2}$$

$$x_i^t - x_i^{t-1} + \sum_{(i,j)\in\Gamma(i)} y_{ij}^{t-1} - \sum_{(j,i)\in\Gamma^{-1}(i)} y_{ji}^{t-1} = 0 \quad \forall i \in C \setminus (C_R \cup C_S), \forall t \in T \tag{3}$$

$$\sum_{(i,j)\in\Gamma(i)} y_{ij}^t - x_i^t \leqslant 0 \quad \forall i \in C \setminus C_S, \forall t \in T \tag{4}$$